Convective Parameterization and its Application in Global Climate Models

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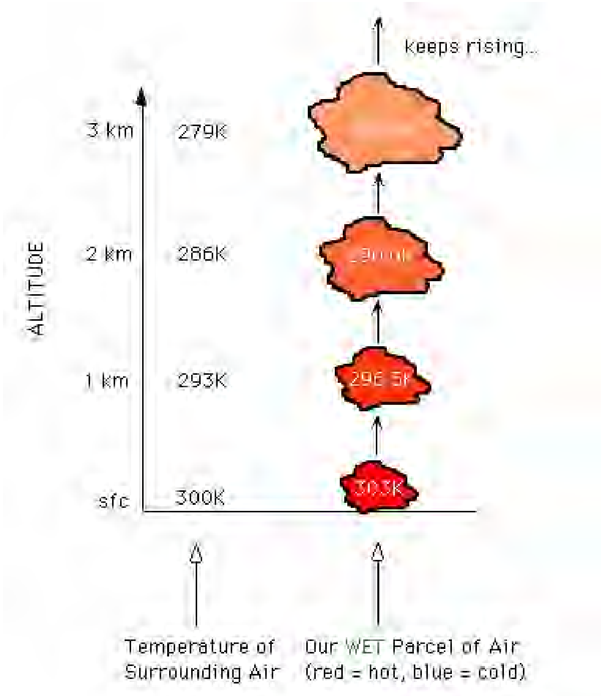
Lecture 1.
Elementary Aspects of Atmospheric Convection and Overview of Convective Parameterization

1. Buoyancy and vertical motion

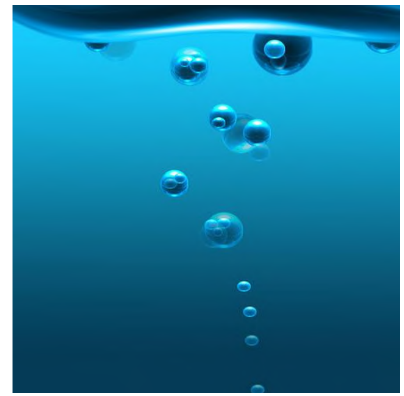
Convection: A thermally direct circulation driven by buoyancy, with strong vertical motion.



The buoyancy of an object relative to its environment is measured by the density differences:

$$B = g\frac{\rho_e - \rho_p}{\rho_e}$$

floating ice



air bubbles in water

Buoyancy as a force acts on the vertical motion of the object. How to relate them? The equation of vertical motion is:

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

P and ρ can be decomposed into horizontally homogeneous mean and perturbation:

$$p = \bar{p} + p' \qquad \rho = \bar{\rho} + \rho' \qquad -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z} = g$$

The vertical velocity equation can be rewritten as:

$$\frac{dw}{dt} = -\frac{1}{\bar{\rho} + \rho'}\frac{\partial(\bar{p} + p')}{\partial z} - g$$

$$= -\frac{1}{\bar{\rho}}\left[1 - \frac{\rho'}{\bar{\rho}} + \left(\frac{\rho'}{\bar{\rho}}\right)^2 + \dots\right]\frac{\partial(\bar{p} + p')}{\partial z} - g$$

$$= -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} - g\frac{\rho'}{\bar{\rho}} + \dots$$

The last term is the buoyancy terms. Making use of the equation of state

$$p = \rho R T \qquad \frac{\rho'}{\bar{\rho}} = \frac{p'}{\bar{p}} - \frac{T'}{\bar{T}}$$

The pressure perturbation term is small compared to the rest for typical atmospheric motion. This can be estimated through scale analysis.

The equation of motion of air within the buoyant air parcel in horizontal (x) direction is:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x}$$

$$\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x}\sim U\frac{\partial U}{\partial x}=\frac{1}{2}\frac{\partial}{\partial x}U^2 \quad\Longrightarrow\quad \frac{p'}{\bar{\rho}}\sim U^2$$

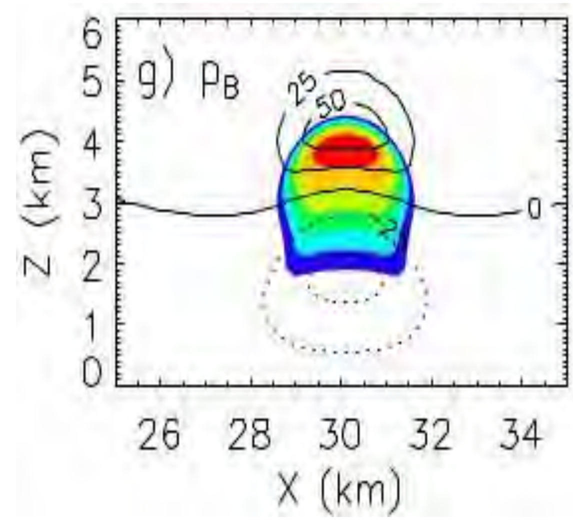
$$\frac{p'}{\bar{p}}\sim\frac{U^2}{R\bar{T}}=\frac{C_p}{C_v}\left(\frac{U^2}{c^2}\right),$$

where $c=\left(\frac{C_p}{C_v}R\bar{T}\right)^{1/2}$ is the speed of sound, which is about 330 to 350 m/s in the atmosphere.

$$\frac{U^2}{c^2}\sim\left(\frac{10}{330}\right)^2\sim 10^{-3} \qquad \frac{T'}{\bar{T}}\sim\frac{3}{300}\sim 10^{-2} \quad\Longrightarrow\quad \frac{\rho'}{\bar{\rho}}=\frac{p'}{\bar{p}}-\frac{T'}{\bar{T}}\approx-\frac{T'}{\bar{T}}$$

$$B = g\frac{T'}{\overline{T}}$$ and the vertical equation of motion becomes:

$$\frac{dw}{dt} = -\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial z} + B$$



Assuming steady-state, and ignoring pressure gradient force:

$$w\frac{\partial w}{\partial z} = \frac{\partial(\frac{1}{2}w^2)}{\partial z} = B \longrightarrow \frac{1}{2}w^2 = \frac{1}{2}w_0^2 + \int B\,dz$$

CAPE

CAPE

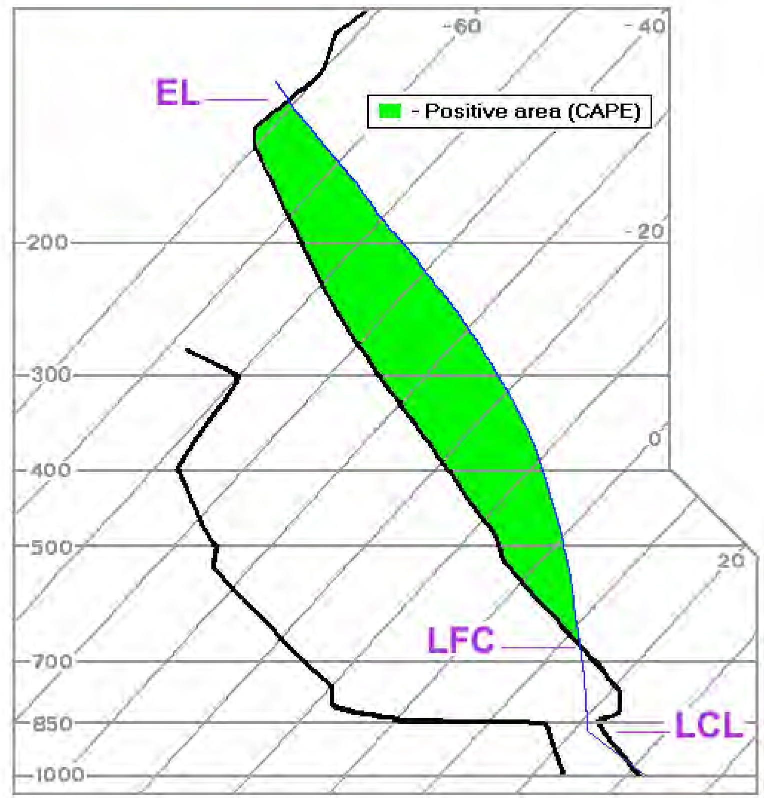
Convective available potential energy (CAPE) is often used to measure the atmospheric stability/instability.

$$CAPE = \int_{init}^{LNB} B\,dz = \int_{init}^{LNB} g\frac{T_{vp} - T_{ve}}{T_{ve}}\,dz$$

$$CAPE = -\int_{p_i}^{p_n} \frac{T_{vp} - T_{ve}}{\rho T_{ve}}\,dp = \int_{p_n}^{p_i} R_d (T_{vp} - T_{ve})\,d\ln p$$

$$w^2 = w_0^2 + 2*CAPE$$

$$w = \sqrt{w_0^2 + 2*CAPE} \approx \sqrt{2*CAPE}$$



2 Moist thermodynamic variables

Mixing ratio r: mass of water vapor per unit mass of dry air,
Specific humidity q: mass of water vapor per unit mass of mixed air including vapor,
Vapor pressure e: partial pressure of water vapor.

Given mixing ratio, how to obtain vapor pressure?

$$r = \frac{\rho_v}{\rho_d} = \frac{e / R_v T}{p_d / R_d T} = \frac{R_d}{R_v}\frac{e}{p - e} = \varepsilon \frac{e}{p - e} \quad \text{where} \quad \varepsilon = \frac{R_d}{R_v} = 0.622$$

$$\begin{aligned} \varepsilon e &= rp - re \\ (\varepsilon + r)e &= rp \end{aligned} \quad \Longrightarrow \quad e = \frac{r}{\varepsilon + r} p = \frac{r / \varepsilon}{1 + r / \varepsilon} p$$

Specific humidity: $$q = \frac{\rho_v}{\rho_d + \rho_v} = \frac{r}{1+r} = \frac{\varepsilon e}{p - e(1-\varepsilon)}$$

Relative humidity: $$RH = \frac{e}{e^*} = \frac{r}{r^*}\left(\frac{1+r^*/\varepsilon}{1+r/\varepsilon}\right)$$

First law of thermodynamics (for unit mass)

$$dQ = dE + p\,d\alpha$$

For a given mass at constant volume, it can be written:

$$(m_v + m_d)dQ = (m_v c_{vv} + m_d c_{vd})dT$$

$$c_v' \equiv \left(\frac{dQ}{dT}\right)_v = \frac{rc_{vv} + c_{vd}}{1+r} = c_{vd}\,\frac{1 + rc_{vv}/c_{vd}}{1+r}$$

$$\approx c_{vd}(1 + rc_{vv}/c_{vd})(1 - r) \approx c_{vd}\left[1 + r(c_{vv}/c_{vd} - 1)\right]$$

$$dQ = c_v' dT + p d\alpha = c_p' dT - \alpha dp$$

$$\alpha = \frac{V}{m_d + m_v} = \alpha_d \frac{1}{1 + m_v / m_d} = \frac{R_d T}{p_d} \frac{1}{1+r} = \frac{R_d T}{p} \frac{p}{p_d} \frac{1}{1+r}$$

$$\frac{p}{p_d} = \frac{p_d + e}{p_d} = 1 + \frac{r}{\varepsilon} \quad \Longrightarrow \quad \alpha = \frac{R_d}{p} T \frac{1 + r/\varepsilon}{1+r} = \frac{R_d}{p} T_v$$

where $$T_v = T \frac{1 + r/\varepsilon}{1+r} \approx T(1 + 0.608r)$$

Virtual temperature: The temperature the air parcel would have reached had the gas constant of the mixed air been the same as that for the dry air with the same density.

Equation of state for moist air is: $$\alpha = \frac{R_d}{p} T_v$$

In an adiabatic process, the first thermodynamic law becomes:

$$c_p^{'} dT = \alpha dp = \frac{T}{p} R_d \frac{1 + r/\varepsilon}{1 + r} dp = R^{'} \frac{T}{p} dp$$

or

$$d \ln T = \frac{R^{'}}{c_p^{'}} d \ln p$$

If r is constant, e.g. in an unsaturated process, the equation can be integrated to give:

$$\theta = T\left(\frac{p_0}{p}\right)^{R^{'}/c_p^{'}} \approx T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_{pd}}(1-0.24r)}$$

For moist adiabatic motion, equivalent potential temperature and moist static energy are conserved.

$$q_e = q\exp\{\frac{L_v r}{c_{pd} T}\}$$

Stability of the atmosphere

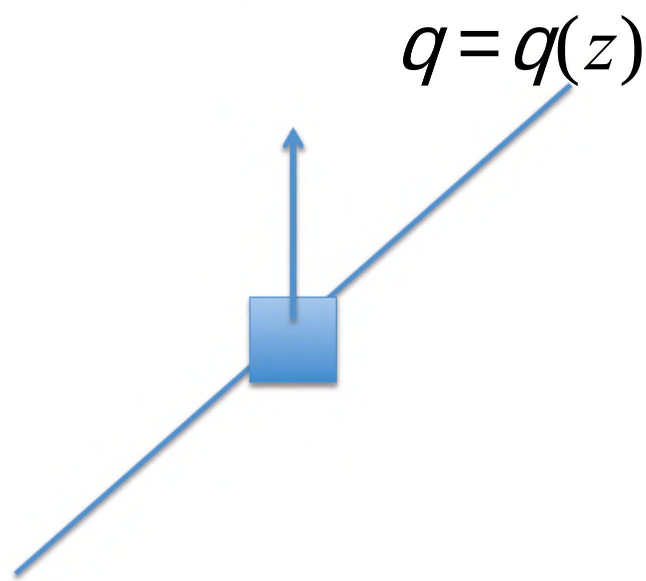
For vertically varying potential temperature $\theta = \theta(z)$, an air parcel at level z is displaced in the vertical by Δz. Since θ is conserved for dry adiabatic motion, the parcel potential temperature remains to be θ(z). The ambient air at the z + Δz level has the potential temperature of

$$\theta(z+\Delta z) = \theta(z) + \frac{\partial \theta}{\partial z}\Delta z$$

Thus, the buoyancy of that parcel with respect to its environment is

$$B = g\frac{\rho - \rho_p}{\rho} = g\frac{T_p - T}{T} = g\frac{\theta_p - \theta}{\theta}$$

$$= g\left(\theta(z) - [\theta(z) + \frac{\partial \theta}{\partial z}\Delta z]\right) / \theta(z) = -\frac{g}{\theta}\frac{\partial \theta}{\partial z}\Delta z$$



$$\frac{dw}{dt} = B = -\frac{g}{q}\frac{\partial q}{\partial z}\mathrm{D}z$$

$$w = \frac{d\mathrm{D}z}{dt} \Rightarrow \frac{d^2\mathrm{D}z}{dt^2} + N^2\mathrm{D}z = 0$$

where $N^2 = \frac{g}{q}\frac{¶q}{¶z}$

Stability for finite vertical displacement

Convective available potential energy (CAPE) is often used to measure the atmospheric stability/instability.

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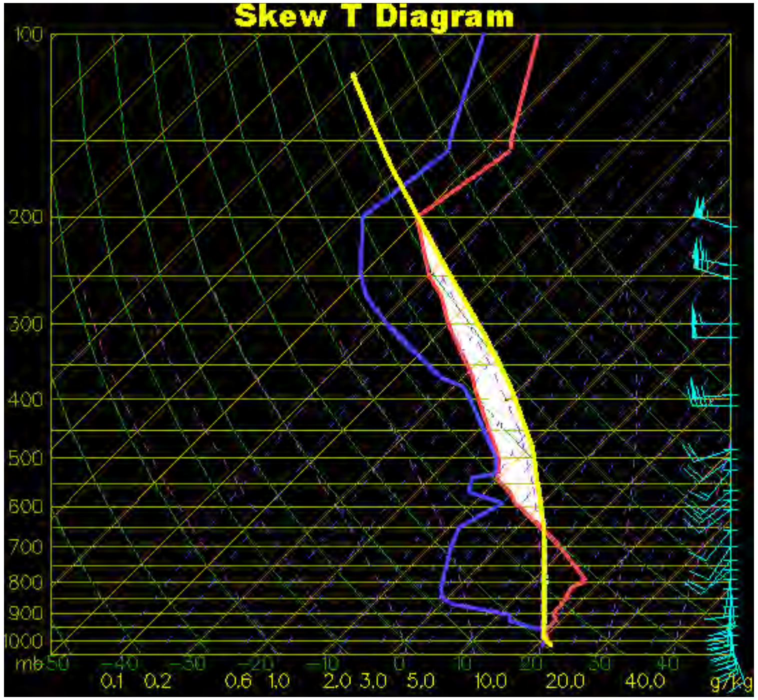


How buoyant is an updraft?

Let’s consider an atmosphere in radiative-convective equilibrium.

Radiative cooling leads T profile to radiative temperature profile with a radiative cooling time scale:

$$\left(\frac{\partial T}{\partial t}\right)_{rad} \approx -\frac{(T - T_{rad})}{\tau_r}$$

Similarly, convective heating adjusts T profile towards convective temperature profile:

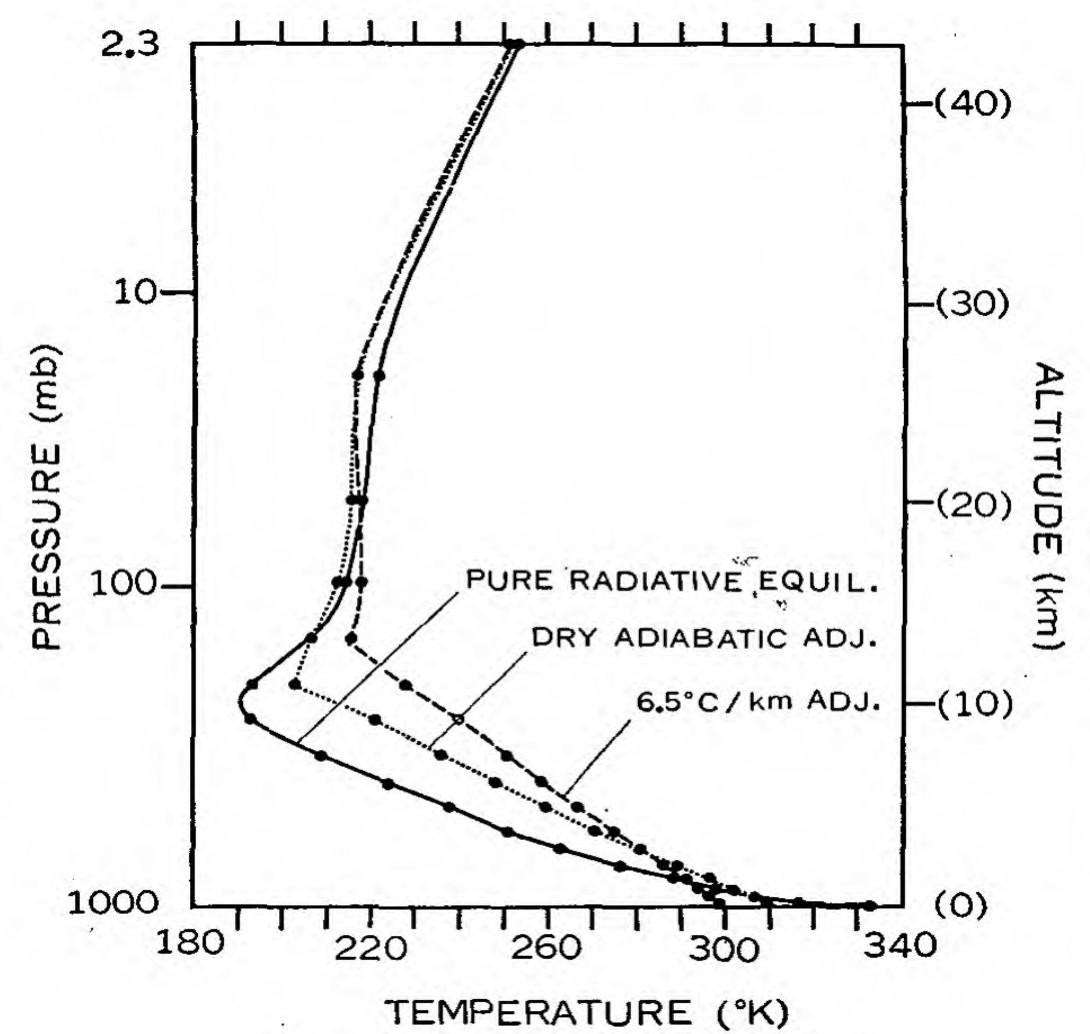
$$\left(\frac{\partial T}{\partial t}\right)_{con} \approx -\frac{(T - T_{con})}{\tau_c}$$

$$\left(\frac{\partial T}{\partial t}\right)_{rad} + \left(\frac{\partial T}{\partial t}\right)_{con} \approx \frac{(T_{rad} - T)}{\tau_r} + \frac{(T_{con} - T)}{\tau_c} \approx 0$$

Solving this for $T\text{-}T_{con}$ gives:

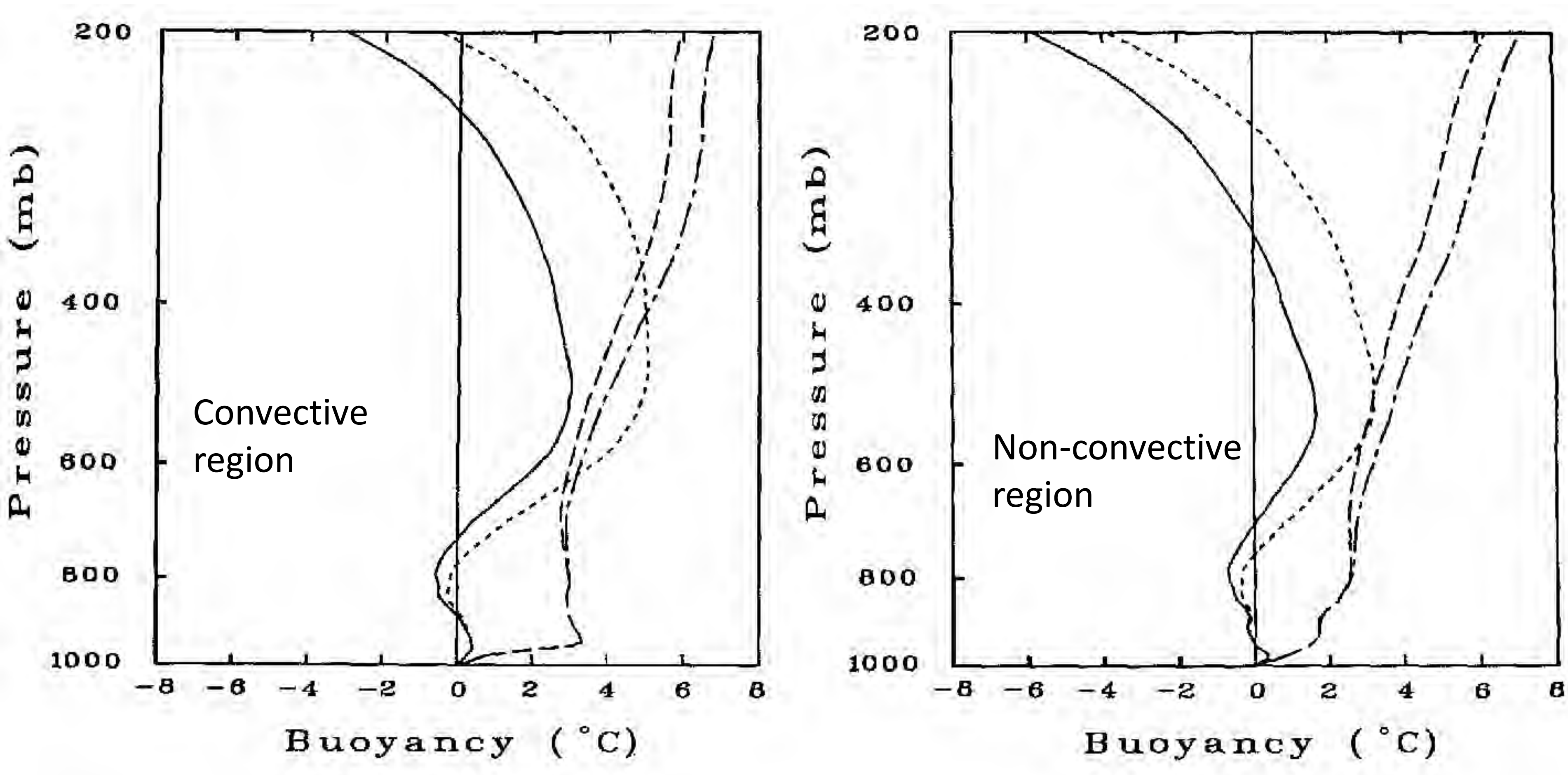
$$T - T_{con} = \frac{T_{rad} - T_{con}}{1 + \tau_r / \tau_c}$$

The different between radiative temperature and convective temperature is about 20 to 40 C: T_{rad}~200K, T_{con}~230K. The timescale for radiative cooling is on the order of 20 days



$$\left(\frac{\partial T}{\partial t}\right)_{rad} \approx -\frac{(T - T_{rad})}{\tau_r} = -2$$

The time scale for convective adjustment is from a few hours to less than a day. So t_{rad}/t_{con} » $20 - 100$. This gives $T_{con} - T \sim 2$ C.



Zhang and McFarlane (1991, MWR)

3. What controls CAPE?

$$CAPE = \int_{init}^{LNB} B\,dz \qquad B = g\frac{\rho - \rho_{parcel}}{\rho} = g\frac{\alpha_{parcel} - \alpha}{\alpha_{parcel}}$$

$$CAPE = \int_{init}^{LNB} g\frac{\alpha_{parcel} - \alpha}{\alpha_{psrcel}}\,dz = \int_{LNB}^{init} (\alpha_{parcel} - \alpha)\,dp$$

Change of CAPE with time:

$$\frac{\partial CAPE}{\partial t} = \frac{\partial}{\partial t}\int_{LNB}^{init} (\alpha_{parcel} - \alpha)\,dp$$

$$= \int_{LNB}^{init} \left(\frac{\partial \alpha_{parcel}}{\partial t} - \frac{\partial \alpha}{\partial t}\right) dp$$

$$+(\alpha_{parcel} - \alpha)\Big|_{p_{init}} \frac{\partial p_{init}}{\partial t} - (\alpha_{parcel} - \alpha)\Big|_{p_{LNB}} \frac{\partial p_{LNB}}{\partial t}$$

For the convenience of derivation, we introduce two new thermodynamic variables: enthalpy and entropy.

enthalpy: $$k = (c_{pd} + \cancel{q_t c_l})T + Lq$$

entropy: $$s = (c_{pd} + \cancel{q_t c_l})\ln T - R_d \ln p + \frac{Lq}{T}$$

$$dk = Tds + \alpha dp$$

$$\left.\frac{\partial k}{\partial s}\right|_p = T(p,s)$$

$$\left.\frac{\partial k}{\partial p}\right|_s = \alpha(p,s)$$

$$\Rightarrow \quad \left.\frac{\partial \alpha}{\partial s}\right|_p = \left.\frac{\partial T}{\partial p}\right|_s = \frac{\partial^2 k}{\partial p \partial s}$$

How is entropy related to some familiar variables?

Dry entropy: $$s_d = c_{pd}\ln T - R_d \ln p = c_{pd}\ln\theta$$

where

$$\theta = T\left(\frac{p_0}{p}\right)^{R_d/c_{pd}}$$

Moist entropy $$s = c_{pd}\ln T - R_d \ln p + \frac{Lq}{T}$$

$$s = c_{pd}\ln\theta_e$$

where

$$\theta_e = \theta\exp\left\{\frac{Lq}{c_{pd}T}\right\}$$

$$\frac{\partial CAPE}{\partial t} = \int_{LNB}^{init} \left(\frac{\partial \alpha_{parcel}}{\partial t} - \frac{\partial \alpha}{\partial t}\right)\Big|_{p} dp$$

The air parcel's specific volume is a function of pressure and moist entropy (or equivalent potential temperature). Thus its change wrt time at a given pressure level is

$$\frac{\partial \alpha_{parcel}}{\partial t}\Big|_{p} = \frac{\partial \alpha_{parcel}}{\partial s_{parcel}}\Big|_{p} \frac{\partial s_{init}}{\partial t} = \frac{\partial T_{parcel}}{\partial p}\Big|_{s} \frac{\partial s_{init}}{\partial t}$$

Remember

$$\frac{\partial \alpha}{\partial s}\Big|_{p} = \frac{\partial T}{\partial p}\Big|_{s}$$

For the environmental air, since it's unsaturated

$$\frac{\partial \alpha}{\partial t}\Big|_{p} = \frac{\partial \alpha}{\partial s_d}\Big|_{p} \frac{\partial s_d}{\partial t} = \frac{\partial T}{\partial p}\Big|_{s_d} \frac{\partial s_d}{\partial t} = \frac{\alpha}{c_p} \frac{\partial s_d}{\partial t}$$

$$\frac{\partial T}{\partial p}\Big|_{s_d} = \frac{\alpha}{c_p}$$ Is dry lapse rate

How to get this relationship?

Hint: Use $\theta = T\left(\frac{p_0}{p}\right)^{R/c_p}$

$$\frac{\partial CAPE}{\partial t} = \int_{LNB}^{init} \left(\frac{\partial \alpha_{parcel}}{\partial t} - \frac{\partial \alpha}{\partial t}\right) dp = \int_{LNB}^{init} \left(\left.\frac{\partial T_{parcel}}{\partial p}\right|_{s} \frac{\partial s_{init}}{\partial t} - \frac{\alpha}{c_p}\frac{\partial s_d}{\partial t} \right) dp$$

$$= (T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t} - \int_{LNB}^{init} \frac{\alpha}{c_p}\frac{\partial s_d}{\partial t}\, dp$$

Entropy for unsaturated air is given by

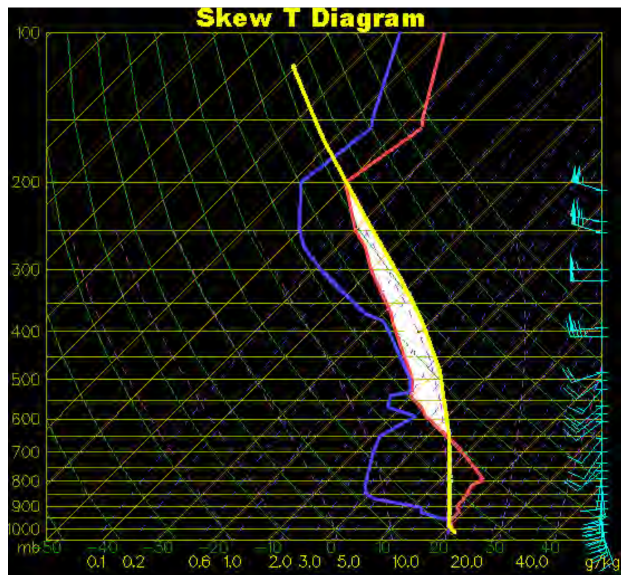
$$s = (c_{pd} + r_t c_l)\ln T - R_d \ln p + \frac{Lr}{T}$$

$$ds_d = c_{pd}\frac{dT}{T} - R_d\frac{dp}{p} = \frac{1}{c_p} d(\ln\theta)$$

$$\frac{\partial s_d}{\partial t} = c_p\frac{\partial \ln\theta}{\partial t} = \frac{c_p}{\theta}\frac{\partial \theta}{\partial t} = \frac{\dot{Q}}{T} - \frac{c_p}{\theta}\left(\mathbf{v}\cdot\nabla\theta + \omega\frac{\partial \theta}{\partial p}\right)$$

$$\frac{\partial CAPE}{\partial t} = (T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t} - \int_{init}^{LNB}\left(\frac{g}{c_p}\frac{\dot{Q}}{T} - \frac{g}{q}(\mathbf{v}\cdot\nabla q + w\frac{\partial q}{\partial p})\right)dz$$

CAPE will increase if 1) parcel initial level (boundary layer) entropy or equivalent potential temperature increases, 2) there is adiabatic or diabatic cooling above the parcel initial level.



$$\frac{\partial CAPE}{\partial t} = (T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t} - \int_{init}^{LNB}\left(\frac{g}{c_p}\frac{\dot{Q}}{T} - \frac{g}{q}(\mathbf{v}\cdot\nabla q + w\frac{\partial q}{\partial p})\right)dz$$

$$\frac{ds_{DB}}{dt} = \frac{gF_s}{p_s - p_t}$$

$$F_{sm} = \frac{1}{T}(SH + LH)$$

For a 100 W/m^2 sensible or latent heat flux applied to a 50 mb (hPa) layer, its resultant CAPE change is:

$$(T_{init} - T_{LNB})\frac{g}{5000}\frac{100}{300} \sim 80\frac{1}{1500} = 0.053 J/kg/s$$ or 4600 J/kg/day.

Over the tropical Pacific Ocean, the latent heat flux is on the order of 100 W/m^2 and sensible heat flux is on the order of 10 W/m^2. So CAPE generation mostly comes from surface latent heat flux. But over land both can reach 300 to 400 W/m^2 in summer during the day.

For a 2 C/day clear sky radiative cooling in the entire troposphere, the CAPE generation is

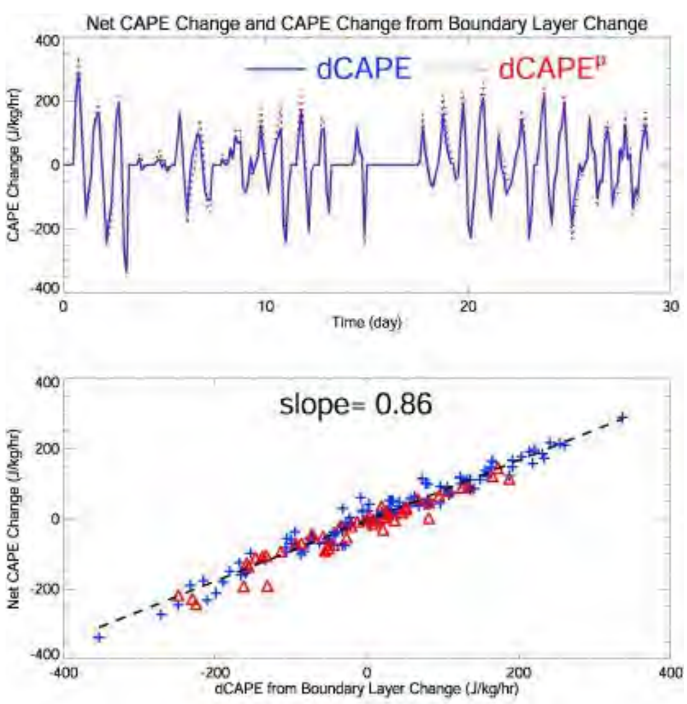
$$\frac{\partial CAPE}{\partial t} = -\int_{init}^{LNB}\left(\frac{g}{c_p}\frac{\dot{Q}}{T}\right)dz = 15000\frac{g}{c_p}\frac{c_p(-2)}{300} \sim 1000$$ J/kg/day.

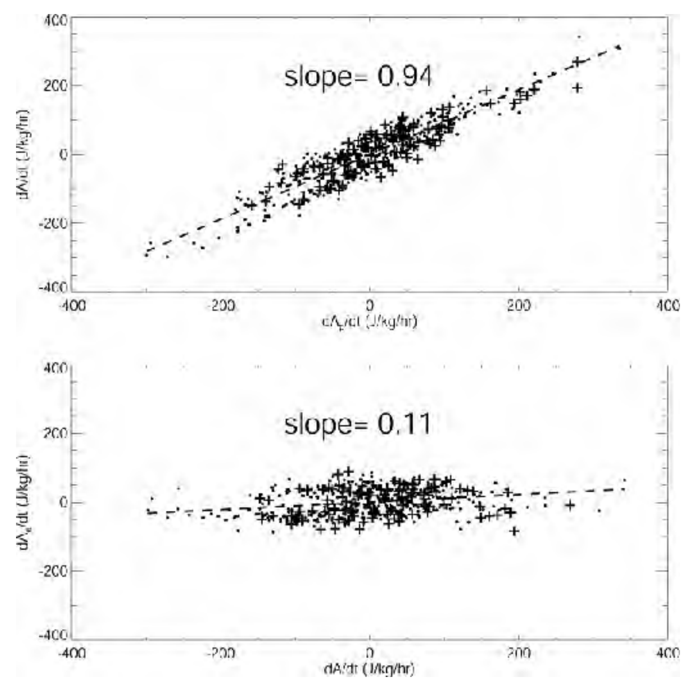
For tropical atmosphere, the actual radiative cooling is < 1C/day, so CAPE generation due to radiative cooling is ~500 J/kg/day or less.

CAPE generation due to adiabatic ascent can be estimated as follows:

$$\frac{\partial CAPE}{\partial t} = \int_{init}^{LNB} \left(\frac{g}{\theta} (\omega \frac{\partial \theta}{\partial p}) \right) dz \sim \frac{10}{320} \frac{50 \sim 100mb}{day} \frac{50K}{900mb} 10^4 m \approx 850 \sim 1700 \quad \text{J/kg/day.}$$

Conclusion: most of the CAPE generation comes from surface sensible and latent heat fluxes.





Relationship between tropical convergence and SST

Before we indulge ourselves into convection parameterization, let's look at the overall effect of convection on tropical circulation. The large-scale temperature and moisture equations can be written as

$$\frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\bar{\mathbf{v}}\, \bar{s}) + \frac{\partial \bar{\omega} \bar{s}}{\partial p} = \bar{Q}_R + L(c-e) - \nabla \cdot (\overline{\mathbf{v}' s'}) - \frac{\partial \overline{\omega' s'}}{\partial p},$$

$$\frac{\partial \bar{q}}{\partial t} + \nabla \cdot (\bar{\mathbf{v}}\, \bar{q}) + \frac{\partial \bar{\omega} \bar{q}}{\partial p} = -(c-e) - \nabla \cdot (\overline{\mathbf{v}' q'}) - \frac{\partial \overline{\omega' q'}}{\partial p}.$$

Where $s = c_p T + gz$ is dry static energy, Q_R radiative heating, c-e condensation. Often $\nabla \cdot \overline{\mathbf{v}' s'}$ is neglected.

Let's also write $Q_R = g \dfrac{\partial F_R}{\partial p}$ and $-\dfrac{\partial \overline{\omega' s'}}{\partial p} = g \dfrac{\partial F_s}{\partial p}$. F_R is radiative flux (positive upward), and F_s is sensible heat flux due to eddy correlation (e.g. convection in the free troposphere and turbulent flux in PBL and surface).

So we rewrite the above equations:

$$\frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\bar{\mathbf{v}}\,\bar{s}) + \frac{\partial \bar{\omega}\,\bar{s}}{\partial p} = g\,\frac{\partial (F_R + F_s)}{\partial p} + L(c-e)$$

$$\frac{\partial L\bar{q}}{\partial t} + \nabla \cdot (\bar{\mathbf{v}} L\,\bar{q}) + \frac{\partial \bar{\omega}\,L\bar{q}}{\partial p} = g\,\frac{\partial F_L}{\partial p} - L(c-e)\,.$$

Assume ω=0 at top and bottom of the atmosphere, and use

$< X >= \dfrac{1}{g}\displaystyle\int_{p_T}^{p_B} X dp$ to represent vertical mass weighted column integral

$$\frac{\partial < s >}{\partial t} + \left\langle \nabla \cdot \mathbf{v}s \right\rangle = (F_R + F_s)\Big|_B - (F_R + F_s)\Big|_T + LP = (F_{RB} - F_{RT}) + SH + LP$$

Similarly, for q the equation becomes

$$\frac{\partial < Lq >}{\partial t} + \left\langle \nabla \cdot \mathbf{v}Lq \right\rangle = F_L\Big|_B - F_L\Big|_T - LP = L(E-P)$$

For long-term average, the time derivative term disappears. Large-scale divergence in convectively active region has a reasonably simple vertical structure of low level convergence and upper level divergence, with a nondivergent level in mid-troposphere.

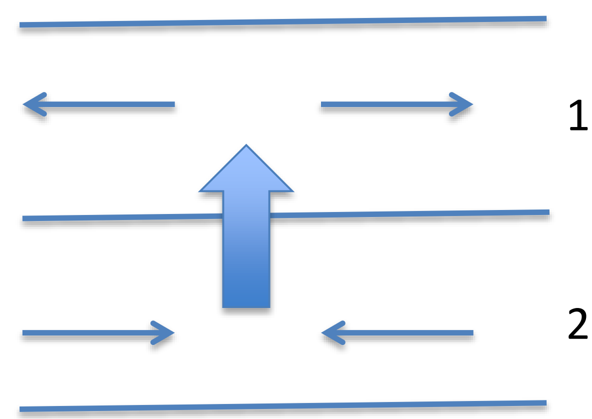
As a first order approximation, we divide the troposphere into two layers (layer 1 for top layer and layer 2 for bottom layer), and define average quantities in each layer:

$$\nabla \cdot \mathbf{v}_2 = \int_{p_M}^{p_B} \nabla \cdot \mathbf{v}\, dp / dp = -\nabla \cdot \mathbf{v}_1$$

$$s_2 = \int_{p_M}^{p_B} s \nabla \cdot \mathbf{v}\, dp / (dp \nabla \cdot \mathbf{v}_2)$$

$$s_1 = \int_{p_T}^{p_M} s \nabla \cdot \mathbf{v}\, dp / (dp \nabla \cdot \mathbf{v}_1)$$

$$\Delta s = s_1 - s_2$$



$$\int_{p_T}^{p_B} \nabla\cdot\mathbf{v}s\frac{dp}{g} = <\mathbf{v}\cdot\nabla s> + \int_{p_M}^{p_B} s\nabla\cdot\mathbf{v}\frac{dp}{g} + \int_{p_T}^{p_M} s\nabla\cdot\mathbf{v}\frac{dp}{g}$$

$$= <\mathbf{v}\cdot\nabla s> + \frac{dp}{g}s_2\nabla\cdot\mathbf{v}_2 - \frac{dp}{g}s_1\nabla\cdot\mathbf{v}_2 = -\frac{dp}{g}\Delta s\nabla\cdot\mathbf{v}_2 + <\mathbf{v}\cdot\nabla s>$$

$$-\frac{dp}{g}\Delta s\nabla\cdot\mathbf{v}_2 + <\mathbf{v}\cdot\nabla s> = F_{RB} - F_{RT} + SH + LP$$

$$-\frac{dp}{g}L\Delta q\nabla\cdot\mathbf{v}_2 + <\mathbf{v}\cdot\nabla Lq> = L(E-P)$$

In the tropics, the horizontal gradient of s is small, the net top-of-atmosphere radiation is small, net surface radiative flux and sensible heat flux are also relatively small compared to latent heating due to precipitation. So

$$-\Delta s \nabla \cdot \mathbf{v}_2 \approx \frac{g}{\delta p} LP$$

This states that export of dry static energy is balanced by latent heat release from precipitation. Since $\Delta s > 0$, it also implies regions of low-level convergence are regions of precipitation.

Similarly, for moisture we have

$$\Delta q \nabla \cdot \mathbf{v}_2 \approx \frac{g}{\delta p} P$$

In both of the above equations, we only retained the largest terms. Since $\Delta s > 0$ and $\Delta q < 0$, if we combine the two equations smaller terms due to surface and top-of-atmosphere need to be included.

$$-\triangle m \nabla \cdot \mathbf{v}_2 = \frac{g}{\delta p}(F_{RB} - F_{RT} + SH + LE)$$

$$\triangle m = \triangle s + L \triangle q$$

Δm is called gross moist stability. In the tropics, the climatological mean flux terms (radiation, sensible and latent heat flux) in general have a fairly smooth horizontal structure. So the sharp gradient in horizontal convergence must mostly come from the horizontal structure of Δm .

$$-\nabla \cdot \mathbf{v}_2 = \frac{g}{\delta p}(F_{RB} - F_{RT} + SH + LE) / \triangle m$$

This is the idea of Neelin's simple tropical circulation model. In regions of small Δm , horizontal convergence is large, and vice versa.

Since the upper troposphere has little moisture Δm can be approximated by

$$\Delta m = \Delta s - Lq_2 .$$

We can also express q_2 in terms of sea surface temperature

$$q_2 = RH * q^*(T_s - \delta T)$$

$$-\nabla \cdot \mathbf{v}_2 = \frac{g}{\delta p}(F_{RB} - F_{RT} + SH + LE) / [\Delta s - L * RH * q^*(T_s - \delta T)]$$

High SST region corresponds to high low-level convergence!

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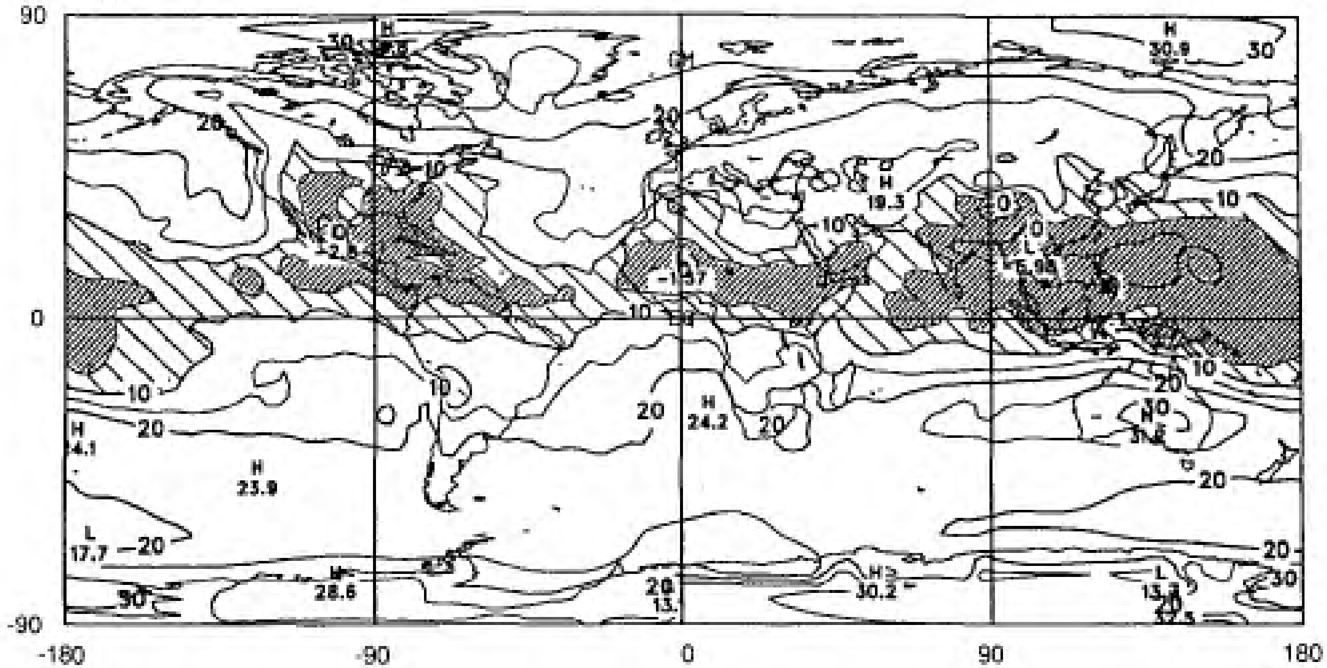
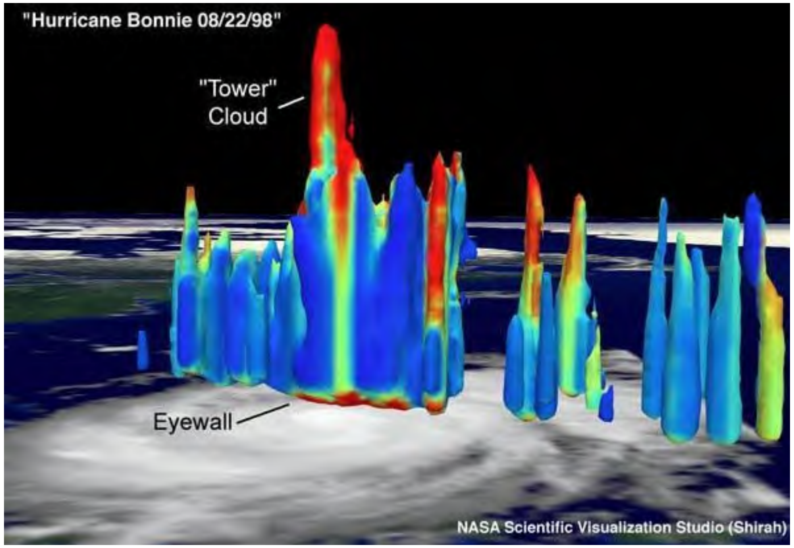


History of Convection Parameterization Development



- Riehl and Malkus: hot towers
- Manabe: moist convective adjustment
- Betts-Miller: soft adjustment
- Kuo: cumulus mixing
- Arakawa and Schubert: mass flux and compensating subsidence

The goal of convective parameterization is to answer the following questions

- Is there going to be convection?
- How much convection?
- What are the properties inside convection?
- How much is the effect of convection (on temperature, moisture, momentum, pollution transport, etc.)?

$$\frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\bar{\mathbf{v}}\,\bar{s}) + \frac{\partial \bar{\omega}\bar{s}}{\partial p} = \bar{Q}_R + L(c - e) + L_d \Upsilon_{ds} + L_f \Upsilon_{fm} - \nabla \cdot (\overline{\mathbf{v}'s'}) - \frac{\partial \overline{\omega' s'}}{\partial p}$$

$$\frac{\partial \bar{q}}{\partial t} + \nabla \cdot (\bar{\mathbf{v}}\,\bar{q}) + \frac{\partial \bar{\omega}\bar{q}}{\partial p} = -(c - e) - \Upsilon_{ds} - \nabla \cdot (\overline{\mathbf{v}'q'}) - \frac{\partial \overline{\omega' q'}}{\partial p}$$

$$C_p \left(\frac{\partial \bar{T}}{\partial t} \right)_c = L(c - e) - \frac{\partial \overline{\omega' s'}}{\partial p}$$

$$\left(\frac{\partial \bar{q}}{\partial t} \right)_c = (e - c) - \frac{\partial \overline{\omega' q'}}{\partial p}$$

The task of convective parameterization is to represent the collective effects of convection on the right hand side in terms of the resolved fields.

Adjustment schemes

Dry adjustment

if $-\frac{\partial T}{\partial p} > \gamma_d$

$$\delta q = 0$$

$$\frac{\partial}{\partial p}\theta(T + \delta T, p) = 0$$

$$\int_{p_B}^{p_T} \delta T \, dp = 0$$

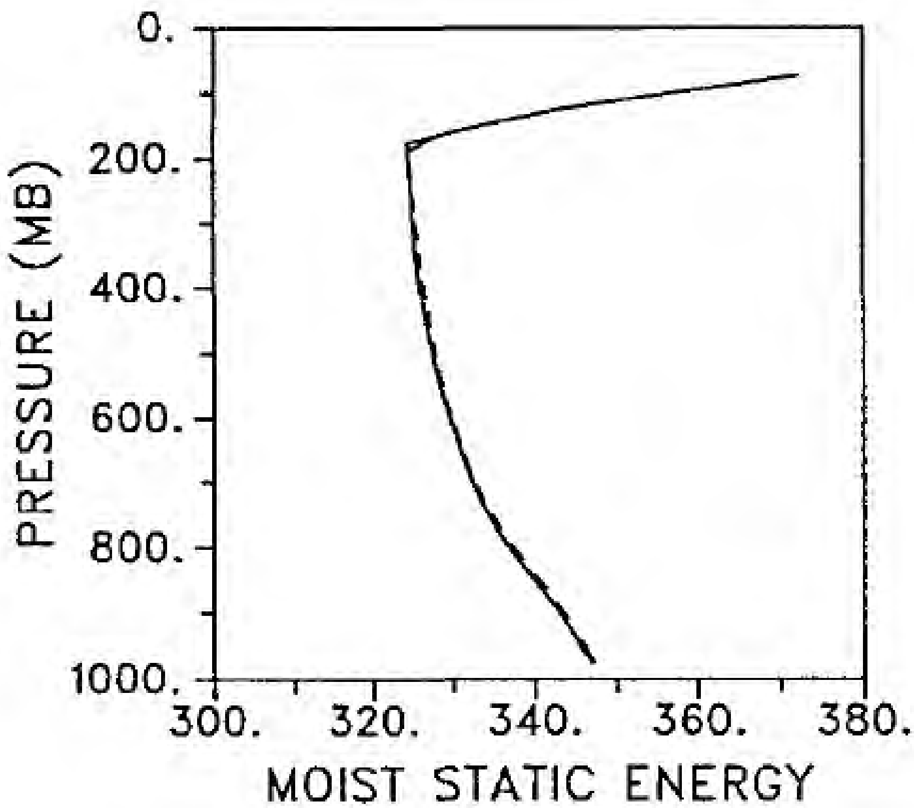
Moist adjustment

if $-\frac{\partial T}{\partial p} > \gamma_m$

$$\frac{\partial}{\partial p}\theta_e(T + \delta T, q + \delta q, p) = 0$$

$$q + \delta q = q_s(T + \delta T, p)$$

$$\int_{p_B}^{p_T} c_p \, \delta T + L \, \delta q \, dp = 0$$



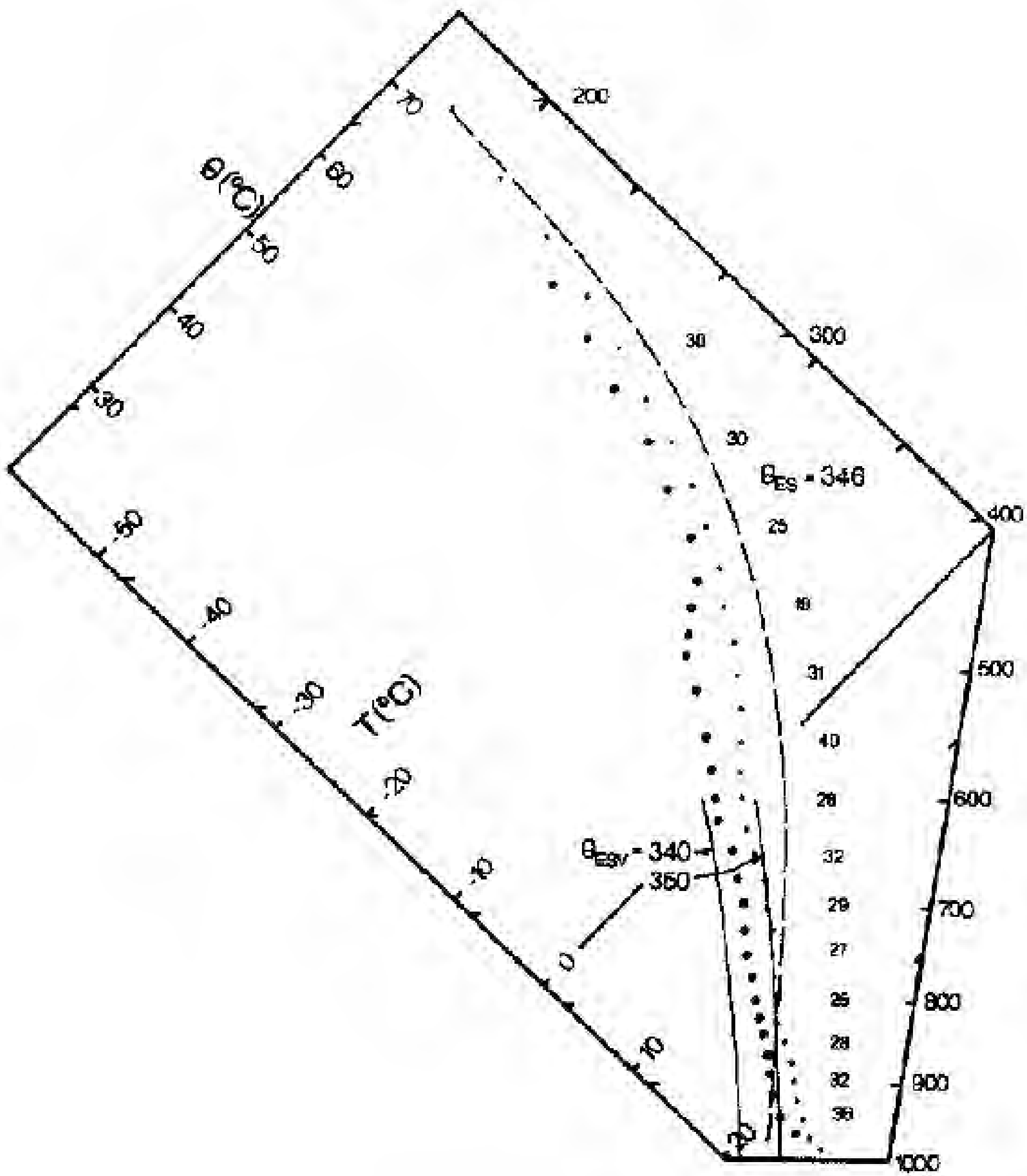
0.
200.
400.
600.
800.
1000.
PRESSURE (MB)
300.
320.
340.
360.
380.
MOIST STATIC ENERGY

Betts-Miller scheme (soft adjustment)

$$\left(\partial T / \partial t\right)_c = (T_r - \bar{T}) / \tau$$

$$\left(\partial q / \partial t\right)_c = (q_r - \bar{q}) / \tau$$

The reference thermodynamic profiles are empirically determined from observations. The reference temperature profile nearly follows a virtual moist adiabat up to the freezing level, and a slightly more stable profile above.



the reference temperature profile is slightly unstable with respect to the virtual moist adiabat of the near surface air

The moisture profile is specified by utilizing the saturation pressure deficit, which is the difference of pressure between a parcel's lifting condensation level and its original level.

The specified temperature and moisture profiles serve as a first guess. The final reference profiles must satisfy certain energy conservation constraints.

$$\int_{p_t}^{p_b} (h_r - h)\,dp = 0$$

The precipitation rate on the surface is just the column integrated moisture change due to this adjustment

$$P = -\frac{1}{g}\int_{p_t}^{p_b} \left(\frac{q_r - q}{\tau} \right) dp$$

In Betts-Miller scheme, the large-scale processes act to pull the temperature and moisture profiles away from the reference profiles and convection acts to push them back to the reference profiles within an adjustment timescale.

Precipitation is a by-product of this scheme. At the same time it also serves as a closure.

If precipitation calculated is positive, the scheme is activated and the ensuing temperature and moisture changes are added to the thermodynamic equations. If the calculated precipitation is negative, convection is not allowed.

Kuo Scheme

The scheme was based on an observational fact that convection is highly correlated with the low-level moisture convergence

It assumes that:

(i) convection occurs in a region where the atmosphere is conditionally unstable and there is low-level moisture convergence;

(ii) convective clouds originate from the boundary layer and the cloud temperature and moisture profiles can be characterized by a pseudo-moist adiabat typical of the boundary layer air; and

(iii) clouds extend from the lifting condensation level of the boundary layer air to the neutral buoyancy level of this air.

$$\int_0^{p_s} \frac{\partial \bar{q}}{\partial t} dp = -Pg - \int_0^{p_s} \nabla \cdot (\bar{\mathbf{v}}\,\bar{q})dp + gF_{LH} = g(M_t - P)$$

where $$M_t = -\frac{1}{g}\int_0^{p_s} \nabla \cdot (\bar{\mathbf{v}}\,\bar{q})dp + F_{LH}$$

$$P = \frac{1}{g}\int_0^{p_s} (c - e)dp$$

Kuo assumed that a small fraction of the moisture supply (bM_t) is used to moisten the atmosphere and the rest of it is precipitated out as rain

$$P = (1 - b)M_t$$

$$\frac{1}{g}\int_0^{p_s} \frac{\partial \bar{q}}{\partial t} dp = bM_t$$

Thus, knowing the total moisture supply M_t, which can be computed from the large-scale fields or GCM output, one can compute the surface precipitation P, thus the vertical integral of latent heating

One needs to know the vertical distribution of the heating in order to determine the effect of condensational heating on temperature field at each GCM level. Kuo (1974) assumed that it is distributed according to the temperature difference between the cloud air following a pseudo-moist adiabat and the environmental air.

$$L(c-e) = \frac{gLP}{p_b - p_t} \frac{T_c - \bar{T}}{< T_c - \bar{T} >}$$

Convective effects include not only latent heating, but also eddy transport

$$C_p \left(\frac{\partial \bar{T}}{\partial t} \right)_c = L(c-e) - \frac{\partial \overline{\omega' s'}}{\partial p}$$

$$\overline{\omega' s'} = \sigma \omega_c (s_c - \bar{s}) = \sigma \omega_c (T_c - \bar{T})$$

$$\frac{\partial s_c}{\partial t} + \omega_c \frac{\partial \bar{s}}{\partial p} = \dot{Q}$$

$$\dot{Q} = -\frac{L}{C_p}\, \omega_c (\partial q_c / \partial p)_{ef}$$

$$\frac{\partial s_c}{\partial t} = \frac{T_c - \bar{T}}{\tau}$$

$$\omega_c = -\frac{T_c - \bar{T}}{\tau} \frac{1}{(\partial h / \partial p)_{ef}}$$

$$\overline{\omega' s'} = -\frac{\sigma}{\tau} \frac{(T_c - \bar{T})^2}{(\partial h / \partial p)_{ef}}$$

The amount of moisture needed to create a convective cloud of fractional area σ and pressure depth (p_b - p_t) with temperature T_c and saturation mixing ratio $q_s(T_c)$ is

$$\frac{\sigma}{g}\int_{p_t}^{p_b}\left[\frac{C_p}{L}(T_c - \bar{T}) + q_s - \bar{q}\right]dp$$

This required amount of moisture for cloud generation must be supplied by the large-scale moisture convergence in time τ

$$\tau M_t = \frac{\sigma}{g}\int_{p_t}^{p_b}\left[\frac{C_p}{L}(T_c - \bar{T}) + q_s - \bar{q}\right]dp$$

$$\frac{\sigma}{\tau} = M_t \Big/ \left(\frac{1}{g}\int_{p_t}^{p_b}\left[\frac{C_p}{L}(T_c - \bar{T}) + q_s - \bar{q}\right]dp\right)$$

Substituting $$\frac{\sigma}{\tau} = M_t \Big/ \left(\frac{1}{g}\int_{p_t}^{p_b}\left[\frac{C_p}{L}(T_c - \bar{T}) + q_s - \bar{q}\right]dp\right)$$

into $$\overline{w's'} = -\frac{\sigma}{\tau}\frac{(T_c - \bar{T})^2}{(\partial h/\partial p)_{ef}}$$

$$\overline{w's'} = -\frac{M_t}{\frac{1}{g}\int_{p_t}^{p_b}\left[\frac{C_p}{L}(T_c - \bar{T}) + q_s - \bar{q}\right]dp}\frac{(T_c - \bar{T})^2}{(\partial h/\partial p)_{ef}}$$

Together with $$L(c - e) = \frac{gLP}{p_b - p_t}\frac{T_c - \bar{T}}{< T_c - \bar{T} >}$$

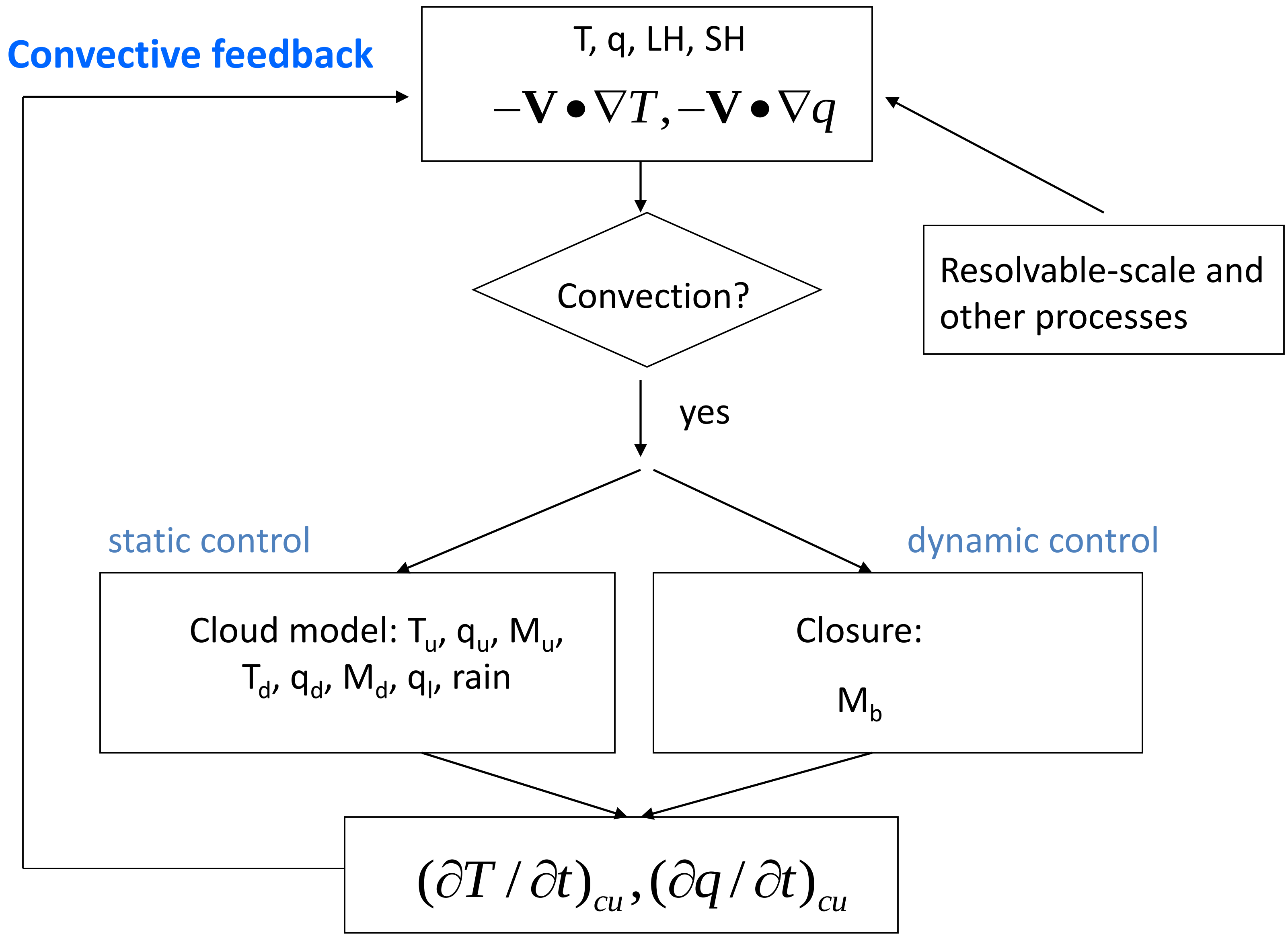
we complete the parameterization

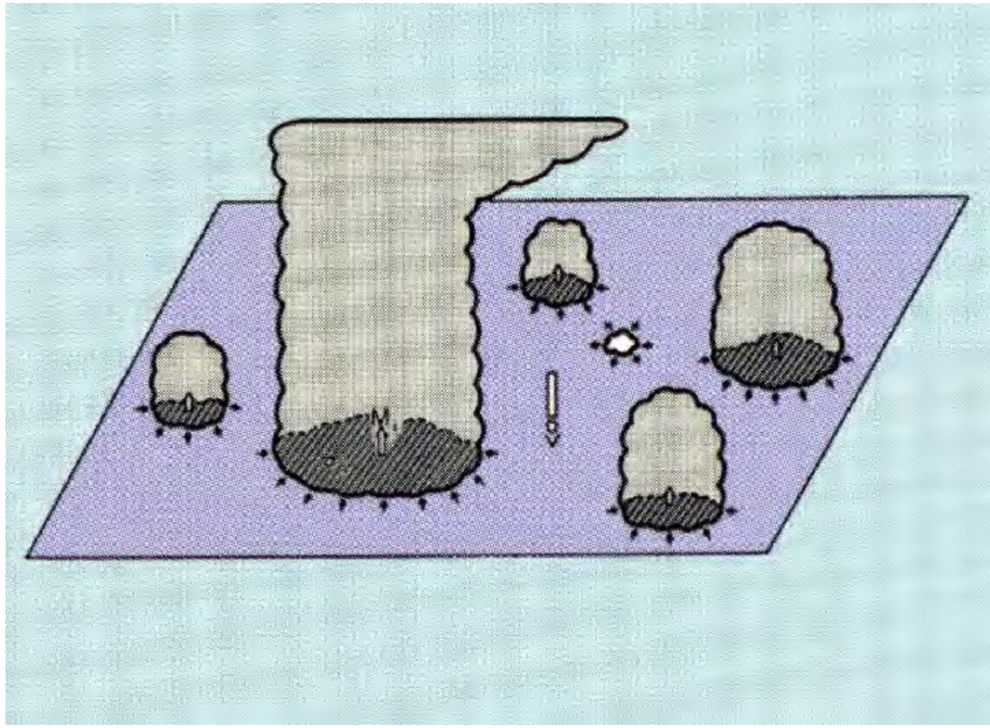
$$C_p\left(\frac{\partial \bar{T}}{\partial t}\right)_c = L(c - e) - \frac{\partial \overline{w's'}}{\partial p}$$

Mass flux schemes

- The mass flux form of convective parameterization has been widely used since Arakawa and Schubert (1974) developed their scheme.
- One important advantage of this type of scheme over others is the ease of incorporating convective transport of tracers, which requires the knowledge of vertical mass flux within convective drafts.
- In addition, detrained mass of hydrometeor from convective cores can be incorporated into large-scale cloud parameterization.
- Most of the parameterization schemes in use nowadays in GCMs are in mass flux form.

Flow chart of convection parameterization





$$\frac{\partial \overline{s}}{\partial t} + \nabla \cdot (\overline{\mathbf{v}}\,\overline{s}) + \frac{\partial \overline{\omega}\,\overline{s}}{\partial p} = \overline{Q}_R + L(c-e) - \nabla \cdot (\overline{\mathbf{v}'s'}) - \frac{\partial \overline{\omega' s'}}{\partial p}$$

$$\frac{\partial \overline{q}}{\partial t} + \nabla \cdot (\overline{\mathbf{v}}\,\overline{q}) + \frac{\partial \overline{\omega}\,\overline{q}}{\partial p} = -(c-e) - \nabla \cdot (\overline{\mathbf{v}'q'}) - \frac{\partial \overline{\omega' q'}}{\partial p}$$

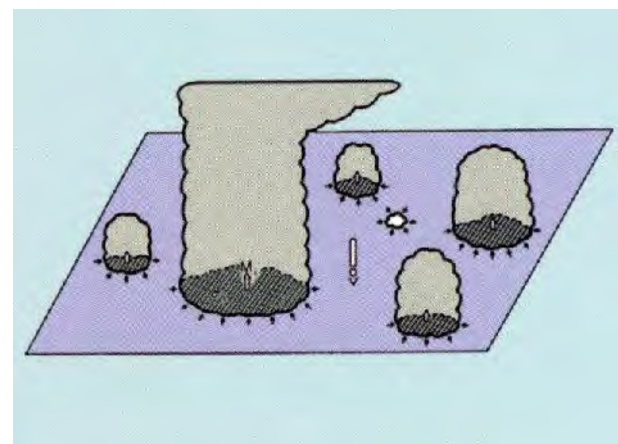
For a given GCM grid box, a variable x can be written as:

$$x = \overline{x} + x'$$

where $\overline{x} = \int x dA$ and $x' = x - \overline{x}$.

Thus, $\overline{x'} = 0$.

Assume that within this area A, there are a number of cumulus clouds occupying a total area of A_c, and the convective free area is $A-A_c$. The mean value of x in convection area is x_c and the mean value in convection free region is x_e.



$$\overline{x} = (A_c x_c + (1- A_c)x_e)/A = a x_c + (1- a)x_e$$

$$x_c' = x_c - \overline{x} = (1- a)(x_c - x_e)$$

$$x_e' = x_e - \overline{x} = - a(x_c - x_e) = - \frac{a}{1- a} x_c'$$

The eddy correlation of the product of two variables x and y then can be written as:

$$\overline{x'y'} = a(x'y')_c + (1-a)(x'y')_e = ax_c'y_c' + (1-a)x_e'y_e'$$

$$= ax_c'y_c' + (1-a)(-\frac{a}{1-a}x_c')(-\frac{a}{1-a}y_c') = \frac{a}{1-a}x_c'y_c'$$

$$= \frac{a}{1-a}(x_c - \bar{x})(y_c - \bar{y})$$

For vertical transport of dry static energy s, let y=s and x=ω:

$$\overline{w's'} = \frac{a}{1-a}(w_c - \bar{w})(s_c - \bar{s})$$

$$\overline{w's'} = \frac{a}{1-a}(w_c - \bar{w})(s_c - \bar{s})$$

If we assume $a \ll 1$ and $\bar{w} \ll w_c$,

$$\overline{w's'} \approx a w_c (s_c - \bar{s}) = -M_c (s_c - \bar{s}) / g$$

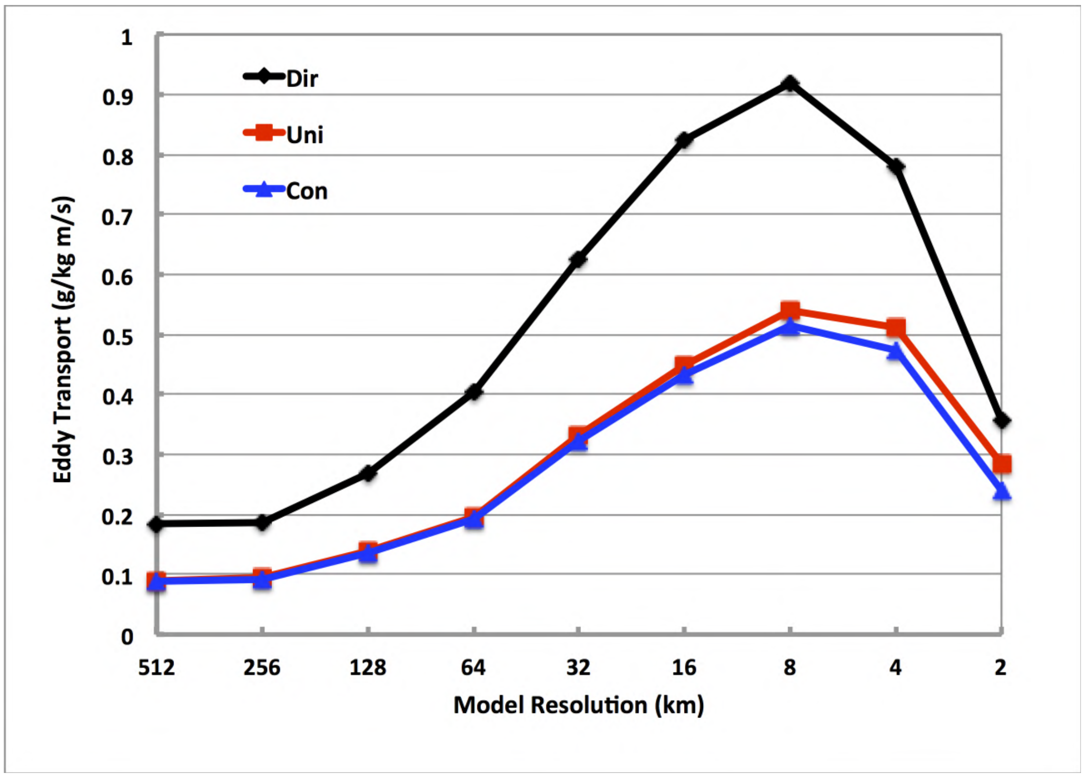
Do we need to assume $a \ll 1$ and $\bar{w} \ll w_c$?

How does it affect convective parameterization if the assumptions are violated, such as in the case of high-res GCM?

$$(w_c - \bar{w}) = w_c - [a w_c + (1-a)\tilde{w}] = (1-a)(w_c - \tilde{w})$$

$$\overline{w's'} \approx a (w_c - \tilde{w})(s_c - \bar{s}) = -M_c (s_c - \bar{s}) / g$$

$$if \qquad \tilde{w} << w_c$$


Dir
Uni
Con
Eddy Transport (g/kg m/s)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
512
256
128
64
32
16
8
4
2
Model Resolution (km)

If both updrafts and downdrafts are considered, the above equation should be written as:

$$\overline{\omega' s'} = -\frac{1}{g}\left[M_u(s_u - \overline{s}) + M_d(s_d - \overline{s})\right]$$

The large-scale temperature and moisture budget equations with the incorporation of the effect of convection can be written (in z coordinates) as:

$$\frac{\partial \overline{s}}{\partial t} + \nabla \cdot (\overline{\mathbf{v}}\,\overline{s}) + \frac{\partial(\overline{r}\,\overline{w}\,\overline{s})}{\overline{r}\partial z} = \overline{Q}_R + L(c - e) + \frac{\partial}{\overline{r}\partial z}[M_u(s_u - \overline{s}) + M_d(s_d - \overline{s})]$$

$$\frac{\partial \overline{q}}{\partial t} + \nabla \cdot (\overline{\mathbf{v}}\,\overline{q}) + \frac{\partial(\overline{r}\,\overline{w}\,\overline{q})}{\overline{r}\partial z} = -(c - e) + \frac{\partial}{\overline{r}\partial z}[M_u(q_u - \overline{q}) + M_d(q_d - \overline{q})]$$