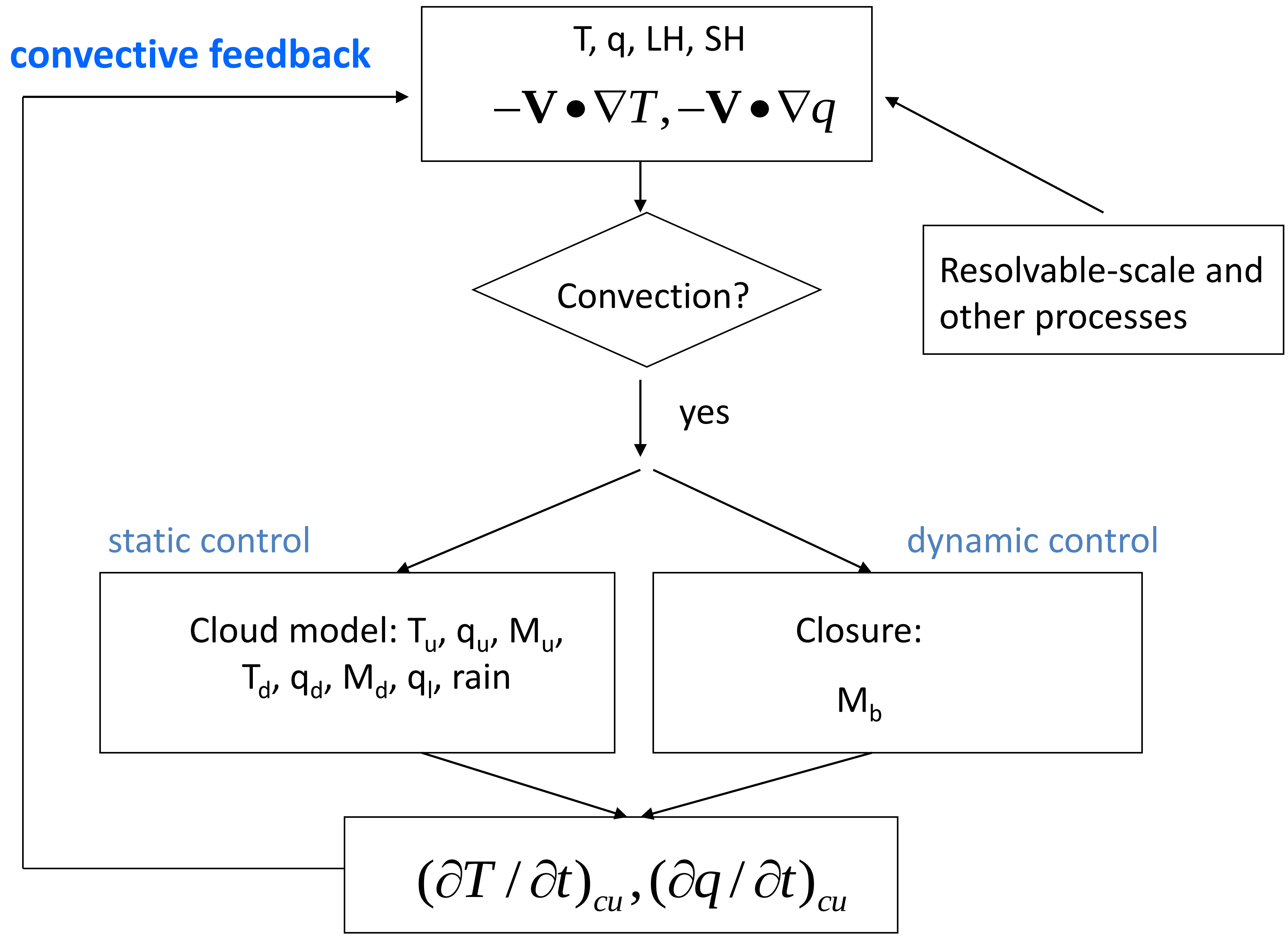
Lecture 2: Trigger functions, 1-D cloud models and closures in convective schemes

Flow chart of convection parameterization



Trigger functions

- Capturing convection at the right time and place is crucial for the realistic simulation of atmospheric variability ranging from weather to climate scales.
- In a convective parameterization scheme, the possibility for convection is assessed based on a set of rules, collectively known as trigger function.
- The trigger function activates the convection parameerization scheme if it detects a potential for deep convection.
- Hence an accurate trigger function is important to the correct simulation of convection.

1. Kain-Fritsch scheme (Kain 2004, used in WRF)
2. Bechtold scheme (Bechtold et al. 2001, used in ECMWF)
3. Tiedtke scheme (Tiedtke 1989, used in ECHAM)
4. Zhang-McFarlane scheme (3 variatants, Zhang-McFarlane 1995, Zhang 2002, Neale et al. 2008, used in NCAR CAM)
5. Donner scheme (Donner 1993, Donner et al. 2001, used in GFDL AM3)
6. Arakawa and Schubert scheme (Arakawa and Schubert 1974, used in NCEP GFS)
7. Modified Tiedtke scheme (Jakob and Siebesma 2003, Bechtold et al. 2004, used in ECHAM)

a. Kain-Fritsch scheme

The trigger function in the Kain-Fritsch scheme (Kain 2004) is determined by the thermodynamic state and large-scale ascent of air in the boundary layer.

For a given sounding of the atmospheric state, the first step is to identify a potential source layer for convection. Beginning from the surface, a 60-mb layer of air is mixed, and its lifting condensation level (LCL) pressure and temperature are calculated.

A temperature perturbation depending on the grid-mean vertical velocity at the LCL is calculated using

$$\delta T = k[w_g - c(z)]^{1/3}$$

where *k* is a constant (k=1) with dimension of K $s^{1/3}$ $cm^{-1/3}$, w_g is the grid-mean vertical velocity (cm s^{-1}), *c(z)* is a threshold vertical velocity given by

$$c(z) = \begin{cases} w_0 (Z_{LCL} / 2000) & Z_{LCL} \le 2000 \\ w_0 & Z_{LCL} > 2000 \end{cases}$$

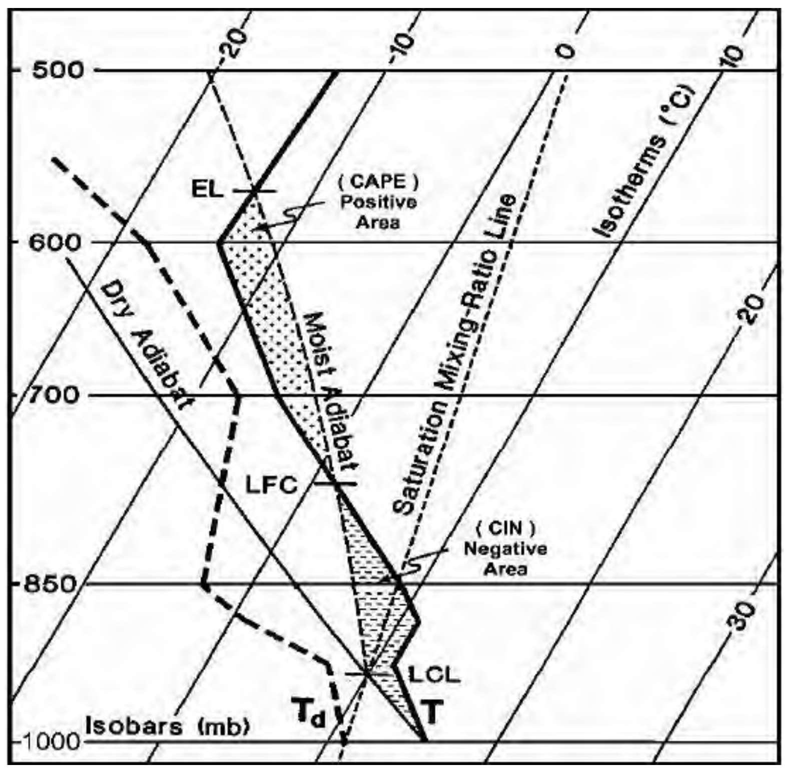
where w_0 = 2 cm s^{-1} and Z_{LCL} is the height of the LCL above the surface in meters.

The temperature perturbation δT is to account for the effect of grid-scale vertical velocity on convection initiation. If $T_{lcl}+\delta T > T_{env}$, the environmental temperature, the layer becomes a candidate for the source layer of convection. An air parcel from this layer is then lifted upward with a vertical velocity of

$$w_{p0} = 1 + 1.1[(Z_{lcl} - Z_{sl})\frac{dT}{T_{env}}]^{1/2}$$

where Z_{sl} is the height at the base of source layer. With this initial vertical velocity, the air parcel is lifted upward with entrainment, which is specified, and water loading.

If the vertical velocity of the parcel remains positive for a minimum depth of 3 km, convection is initiated. Otherwise, the procedure is repeated by moving up one model layer. The process continues until a convective source layer is found or the search has moved up above the lowest 300 mb of the atmosphere, where the search is terminated.



b. Bechtold scheme

The trigger function in the Bechtold scheme (Bechtold et al. 2001) is modified from that of the Kain-Fritsch scheme, with the following changes: The temperature perturbation is related to grid-scale upward motion in a different form:

$$dT = sign(w)c_w\left|w\right|^{1/3}$$

where c_w = 6 K $m^{-1/3}$ $s^{1/3}$, and w is the grid-scale vertical velocity. The cloud top is determined by the neutral buoyancy level (LNB) as opposed to the zero vertical velocity level invoking vertical velocity equation for a lifted parcel.

convection is initiated when the parcel raised from the source layer satisfies the instability criteria at the LCL and the estimated cloud height is at least 3 km.

c. Tiedtke scheme

The Tiedtke scheme (Tiedtke, 1989) is a widely used convection scheme by the European modeling community. It assumes that convection will initiate if the atmospheric column has a net moisture convergence and the surface air is buoyant when lifted to the lifting condensation level

$$\int_0^{p_s} \left(-\mathbf{V} \cdot \nabla q - w\frac{\partial q}{\partial p} \right) dp > 0,$$

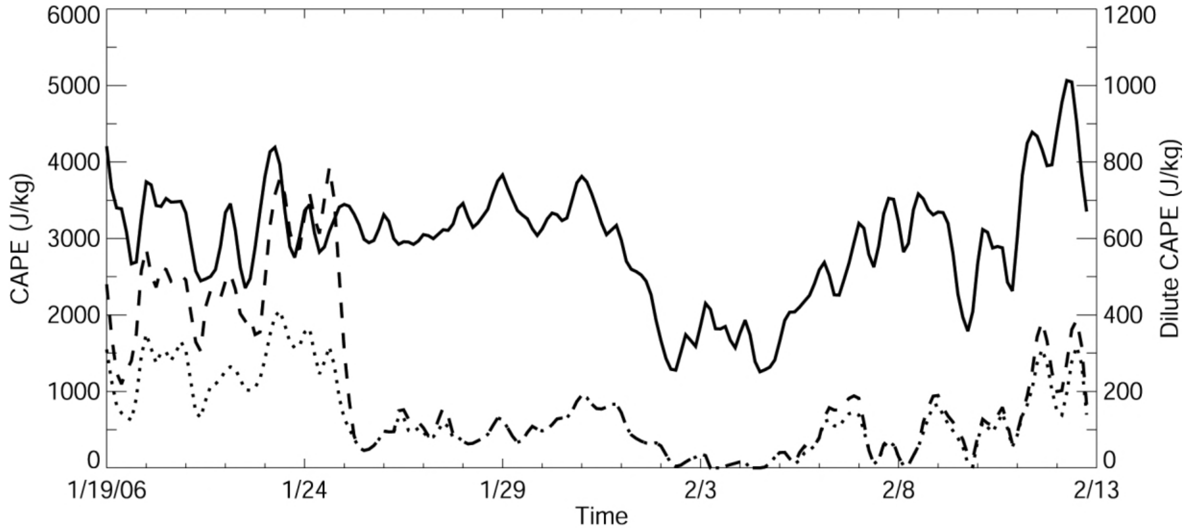
$$T_{vp} + \Delta T > T_{ve} \quad \text{at LCL}$$

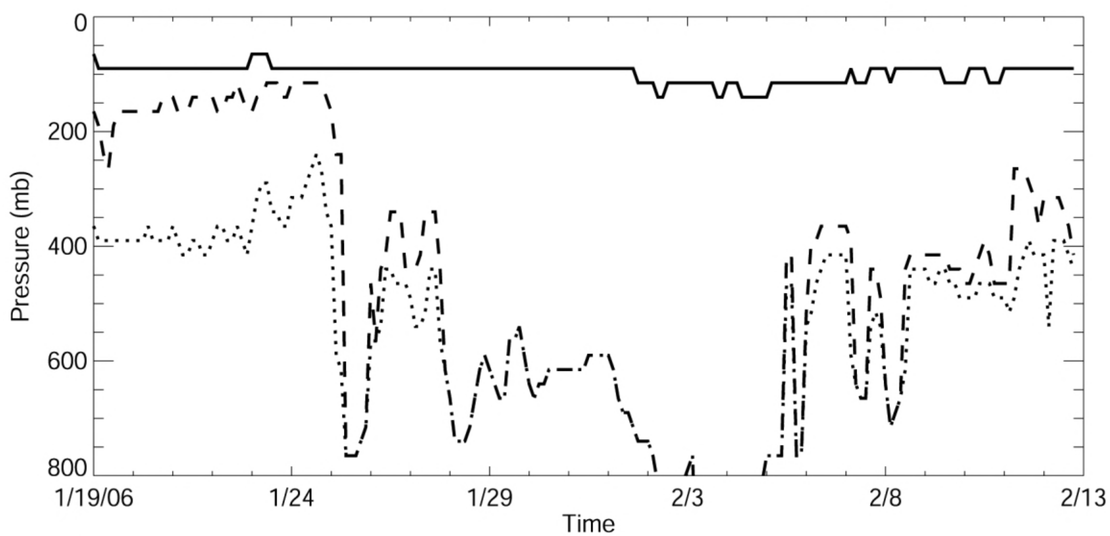
where $\Delta T = 0.5$ K. It also requires that the cloud thickness determined by the difference between the neutral buoyancy level and the LCL be greater than 3 km.

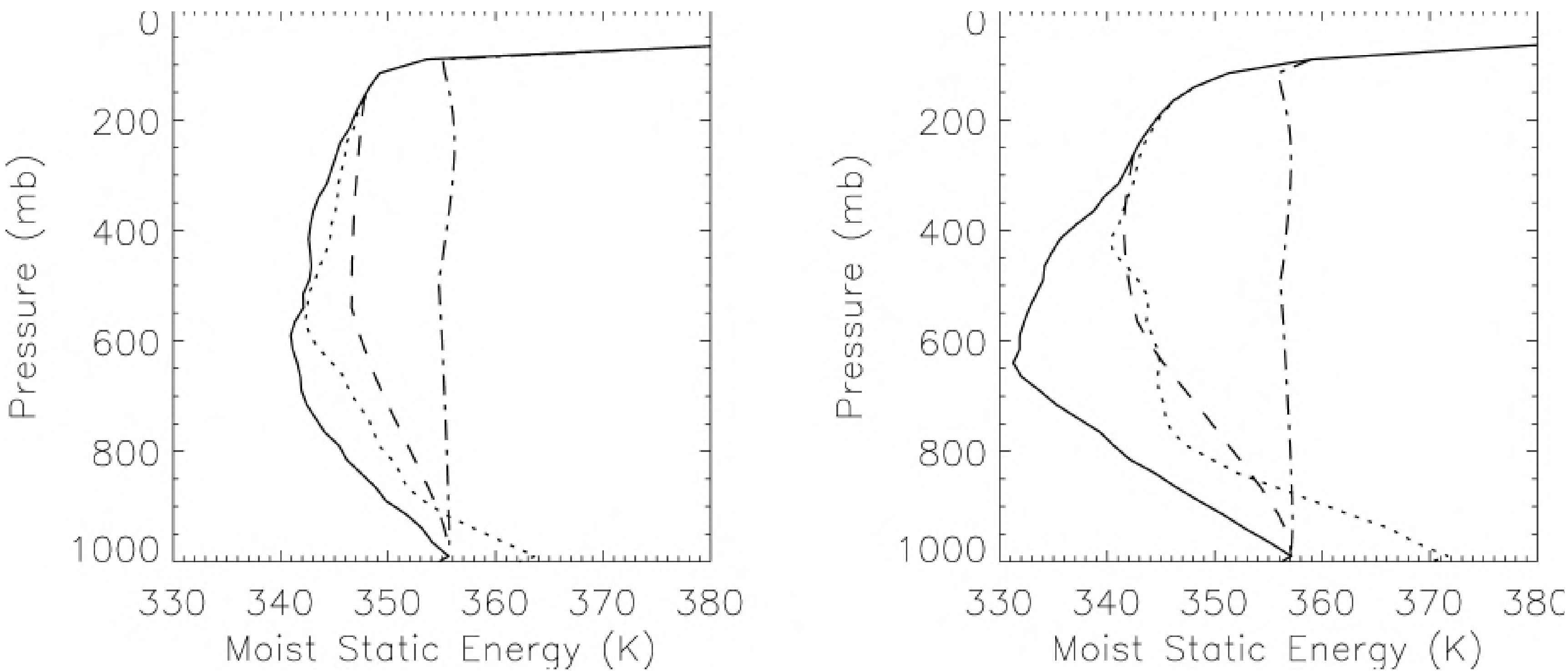
d. Zhang-McFarlane scheme

CAPE is used for convection trigger. It assumes that convection will initiate if an air parcel lifted adiabatically from the level of highest moist static energy below 600 hPa is convectively unstable with CAPE exceeding a threshold value (70 J/kg). Typically, this level is within the lowest couple of model layers.

Recently, Neale et al. (2008) modified the Zhang-McFarlane scheme by including the effect of entrainment dilution in CAPE calculation, and the dilute CAPE exceeding the threshold value of 70 J/kg is used for convection to initiate. This revised trigger (hereafter dilute CAPE) is currently used in the NCAR CAM5.


CAPE (J/kg)
Dilute CAPE (J/kg)
Time
1/19/06
1/24
1/29
2/3
2/8
2/13
0
1000
2000
3000
4000
5000
6000
200
400
600
800
1000
1200


Pressure (mb)
Time
1/19/06
1/24
1/29
2/3
2/8
2/13
0
200
400
600
800



0
200
400
600
800
1000
Pressure (mb)
330
340
350
360
370
380
Moist Static Energy (K)
0
200
400
600
800
1000
Pressure (mb)
330
340
350
360
370
380
Moist Static Energy (K)

Another modification of the Zhang-McFarlane scheme was used to improve the simulation of MJO and ITCZ. In this version, the use of CAPE is replaced by the CAPE generation rate from large-scale forcing in the free troposphere (dCAPE). dCAPE is defined as the amount of CAPE generated by the large-scale advective forcing during a time interval and is calculated by

$$dCAPE = [CAPE(T + advT \times \delta t, q + advq \times \delta t) - CAPE(T, q)] / \delta t$$

where T and q are temperature and moisture and $advT \cdot \delta t$ and $advq \cdot \delta t$ are the temperature and moisture increments by large-scale advection over a time period δt.

In addition, we also consider dilute dCAPE by including entrainment effect on CAPE generation by large-scale advection.

The entrainment rate is the same as that used in Neale et al. (2008) for dilute CAPE calculation.

Therefore, in all there are four variants (CAPE, dilute CAPE, undilute dCAPE and dilute dCAPE) of the trigger functions in the Zhang-McFarlane scheme.

e. Donner scheme

In the Donner scheme convection trigger utilizes cumulative information of vertical velocity at the convection initiation level, and requires that the large-scale vertical velocity integrated over a time span be able to lift the parcel to the level of free convection (LFC):

$$I = \int_{t_0}^{t_1} \omega(p_{init})\,dt \le p_{lfc} - p_{init}$$

where t_0 is the time when ω (p-velocity) at the convection initiation level, chosen to be the first model level above the surface, starts to be negative (upward motion). It also requires that convective inhibition (CIN) is small, CIN < 10 J/kg.

f. Arakawa and Schubert Scheme

Various versions of the Arakawa and Schubert (1974) scheme are used in the NASA Global Modeling Assimilation Office GEOS-5 GCM, GFDL AM2 GCM and NCEP operational Global Forecast System (GFS) model

In most versions of the Arakawa-Schubert scheme a threshold cloud work function is used as the deep convection triggering criteria, which is similar to dilute CAPE.

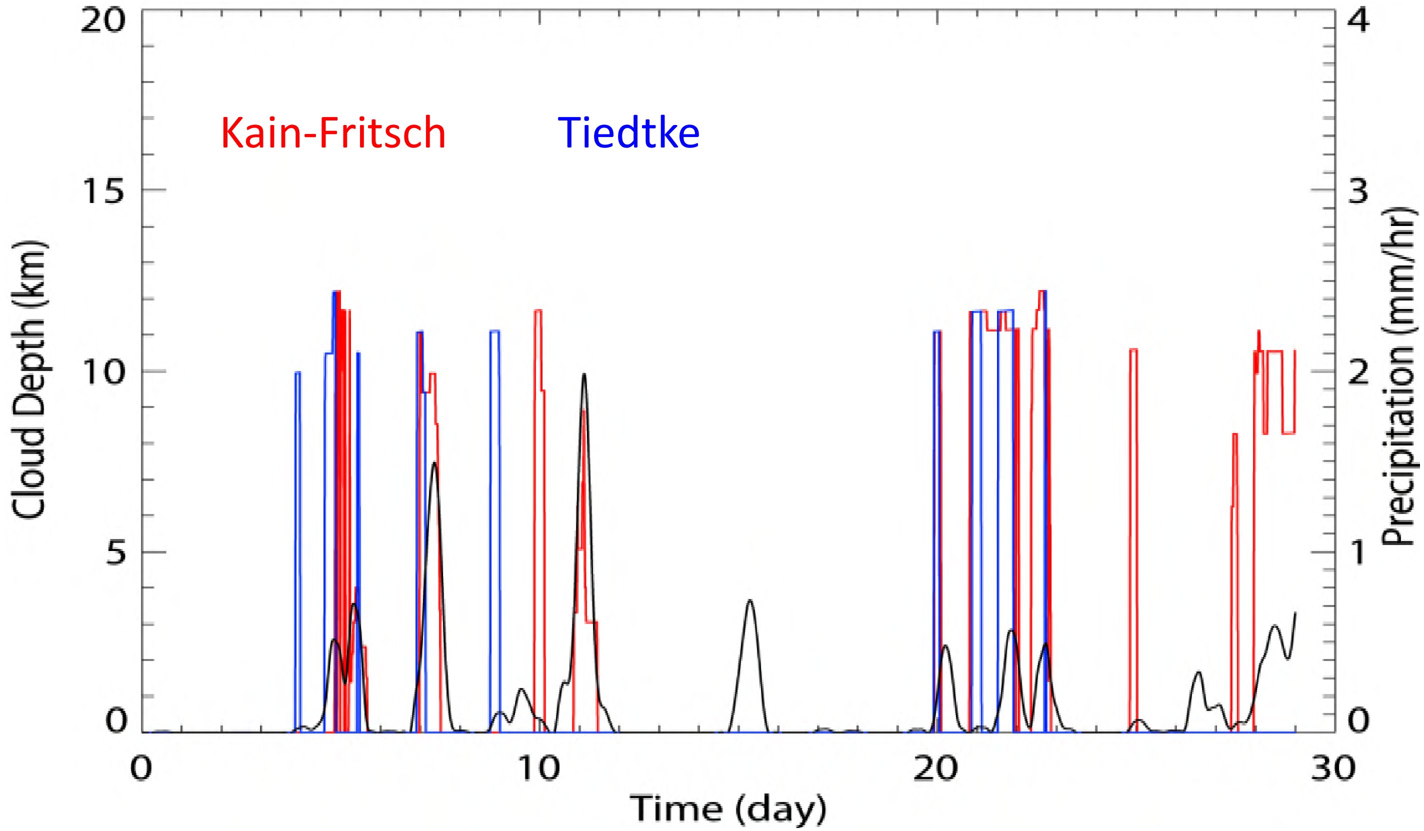
While the GFDL and NASA model use a fixed threshold value of cloud work function as triggering condition, the triggering condition in the NCEP model is that a parcel lifted from the level of maximum moist static energy between the surface and 700 hPa must reach its LFC within 150 hPa of ascent.

In the GFDL model, the lifting condensation level of the surface air is defined as convection base.

The NASA model uses the second lowest model level as cloud base, and the lifted air parcel has the average temperature and moisture of the lowest two model layers.

Data

- Atmospheric System Research SGP’97 IOP (June 19-July 18, 1997)
- Midlatitude Continental Convective Cloud Experiment IOP (MC3E, April 22-June 1, 2011)
- TWP-ICE IOP (Jan. 22-Feb. 12, 2006)
- SGP long-term SCM forcing data (1999-2009)
- TWP long-term data (3 wet seasons, 2004-2007)



Kain-Fritsch
Tiedtke
Cloud Depth (km)
Precipitation (mm/hr)
Time (day)
20
15
10
5
0
4
3
2
1
0
0
10
20
30

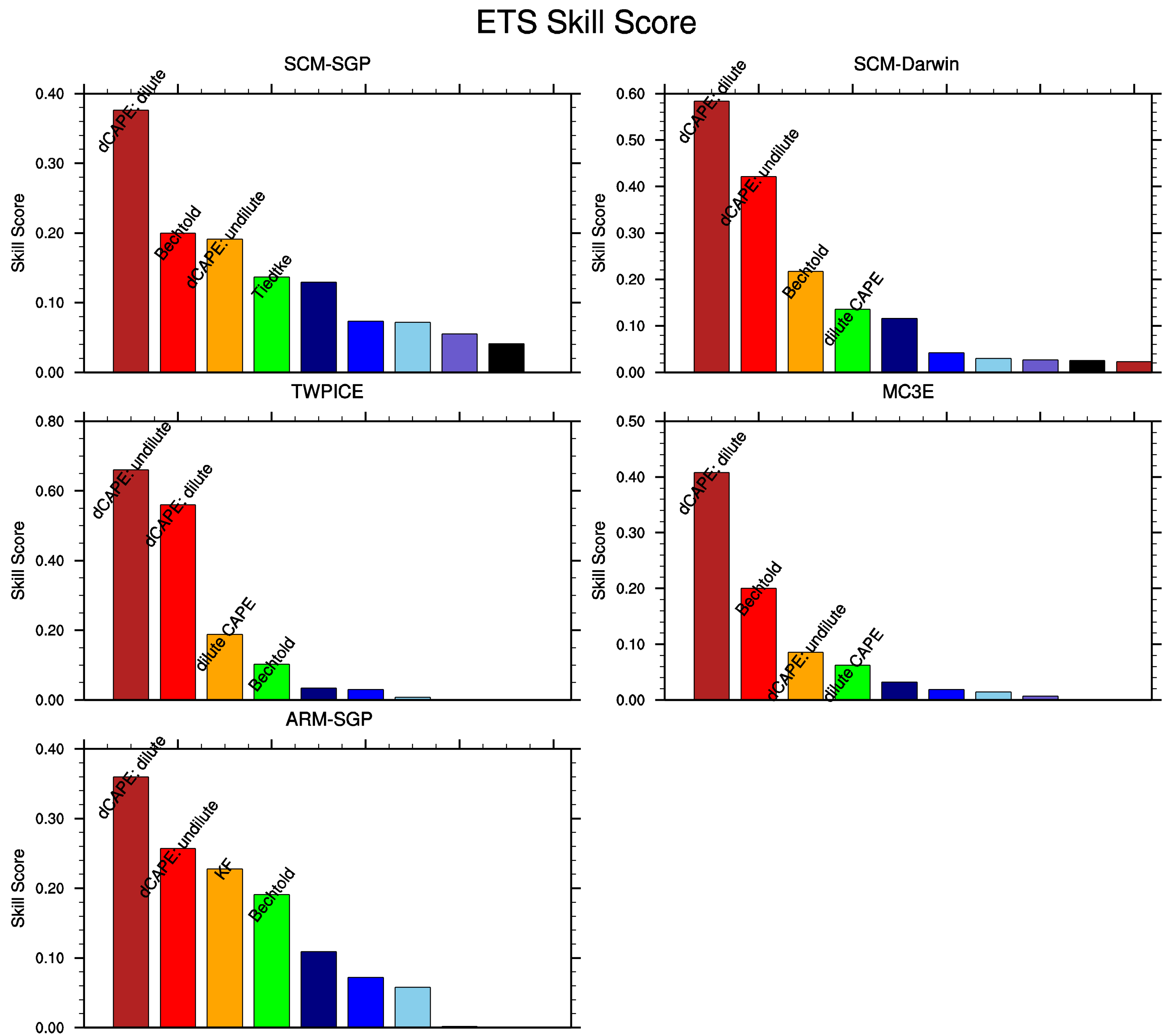
Statistical Evaluation Method

Divide trigger function output into four categories:

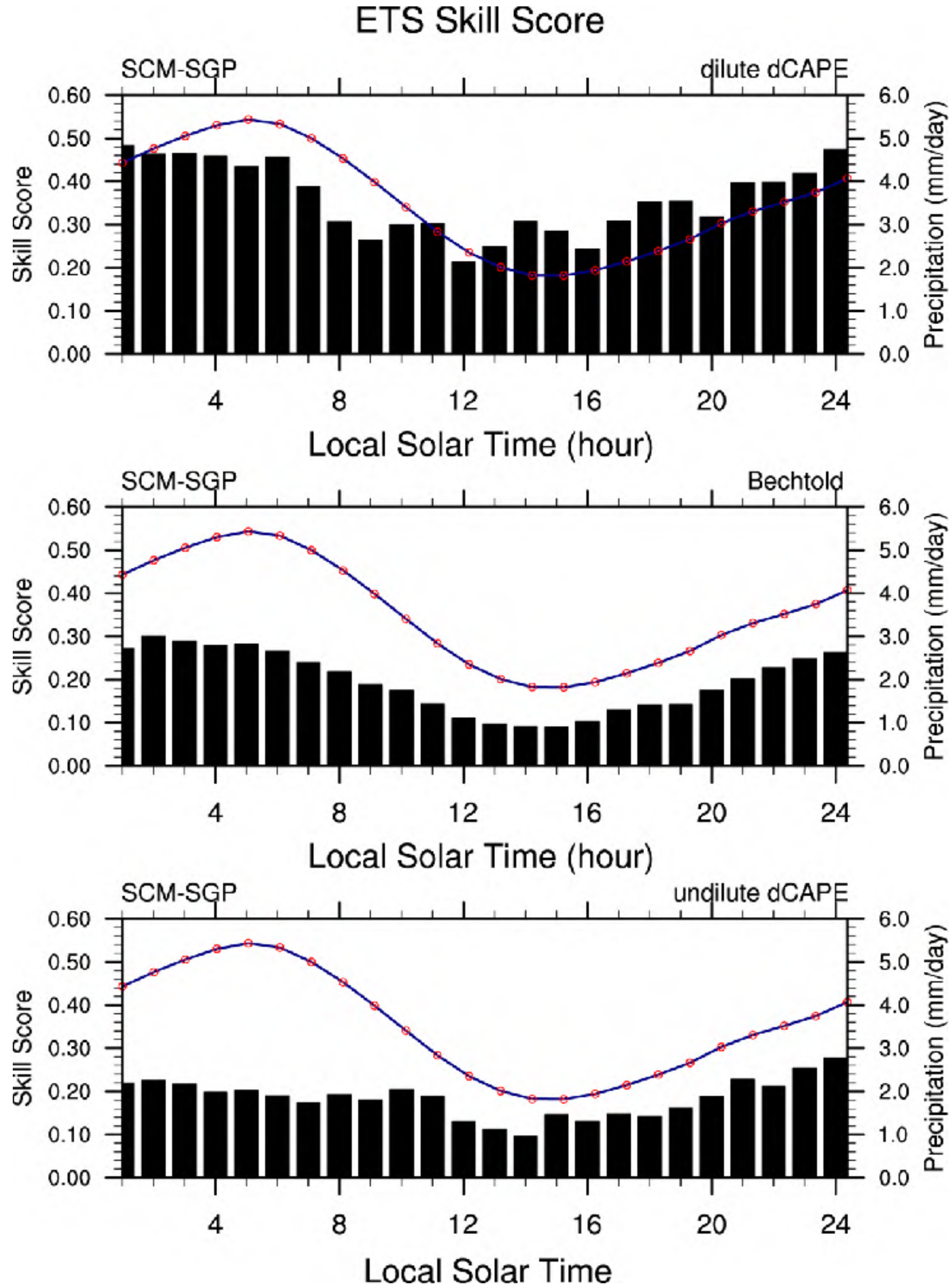
a. Correct prediction of convection onset
b. Over-prediction (trigger predicts convection, but observation shows no convection
c. Under-prediction
d. Correct prediction of no convection

Equitable Threat Score:

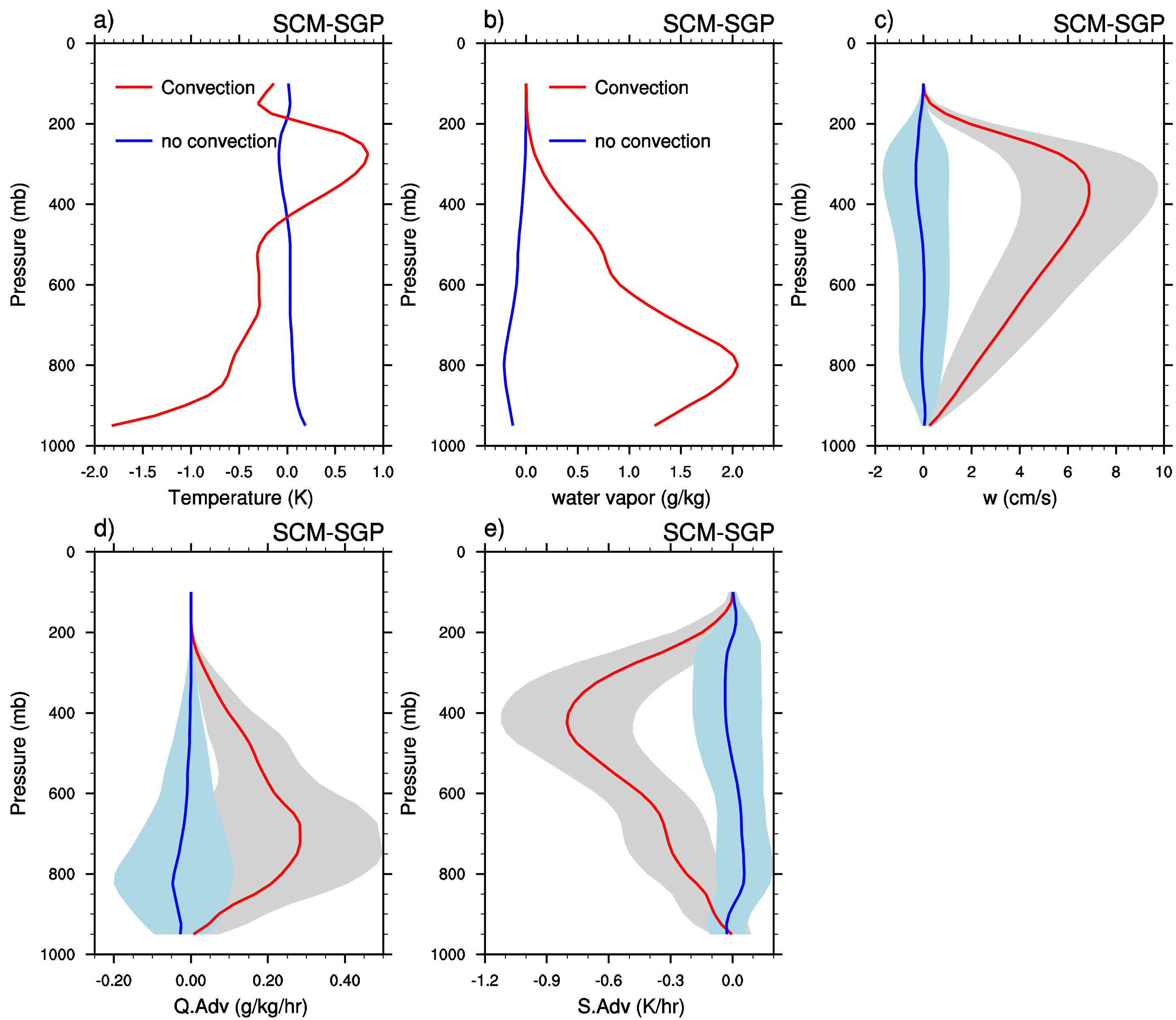
$$ETS = \frac{a-(a+b)(c+d)/n}{a+b+c-(a+b)(c+d)/n}$$



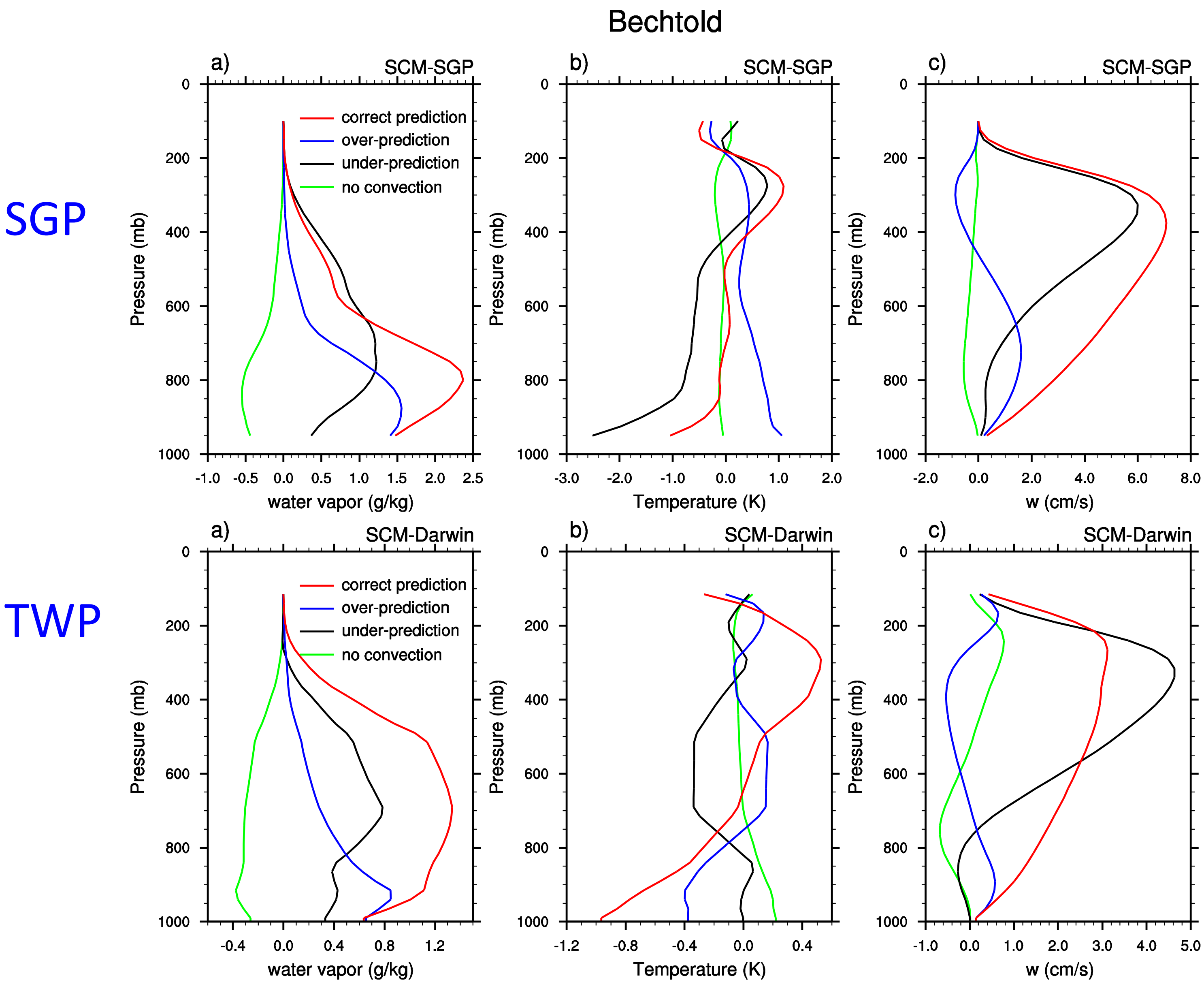
ETS Skill Score
SCM-SGP
SCM-Darwin
TWPICE
MC3E
ARM-SGP
Skill Score
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.80
dCAPE: dilute
dCAPE: undilute
Bechtold
Tiedtke
dilute CAPE
KF



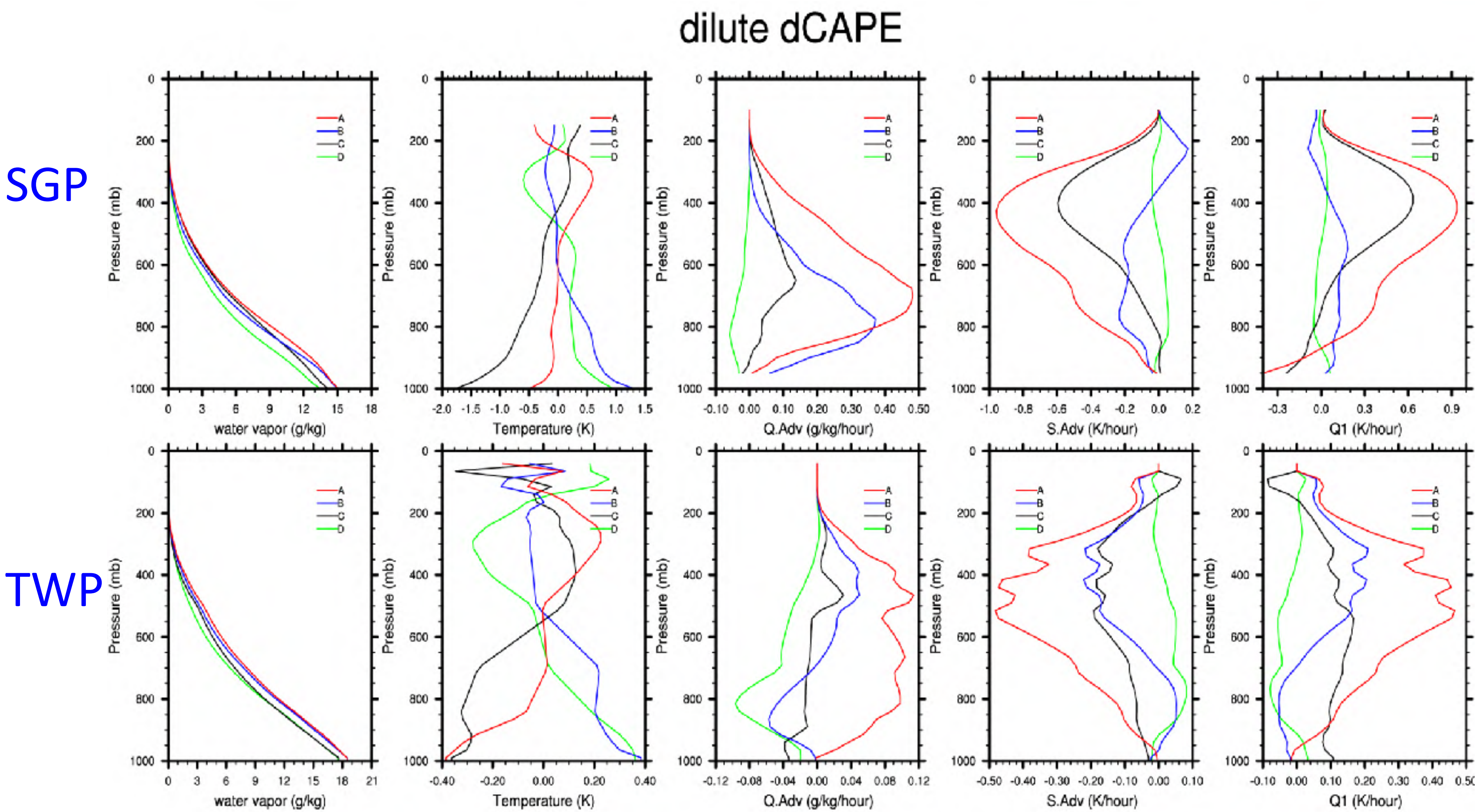
ETS Skill Score
SCM-SGP
dilute dCAPE
Skill Score
Precipitation (mm/day)
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.0
1.0
2.0
3.0
4.0
5.0
6.0
4
8
12
16
20
24
Local Solar Time (hour)
SCM-SGP
Bechtold
Local Solar Time (hour)
SCM-SGP
undilute dCAPE
Local Solar Time


a)
SCM-SGP
Convection
no convection
Pressure (mb)
Temperature (K)
b)
SCM-SGP
Convection
no convection
Pressure (mb)
water vapor (g/kg)
c)
SCM-SGP
Pressure (mb)
w (cm/s)
d)
SCM-SGP
Pressure (mb)
Q.Adv (g/kg/hr)
e)
SCM-SGP
Pressure (mb)
S.Adv (K/hr)

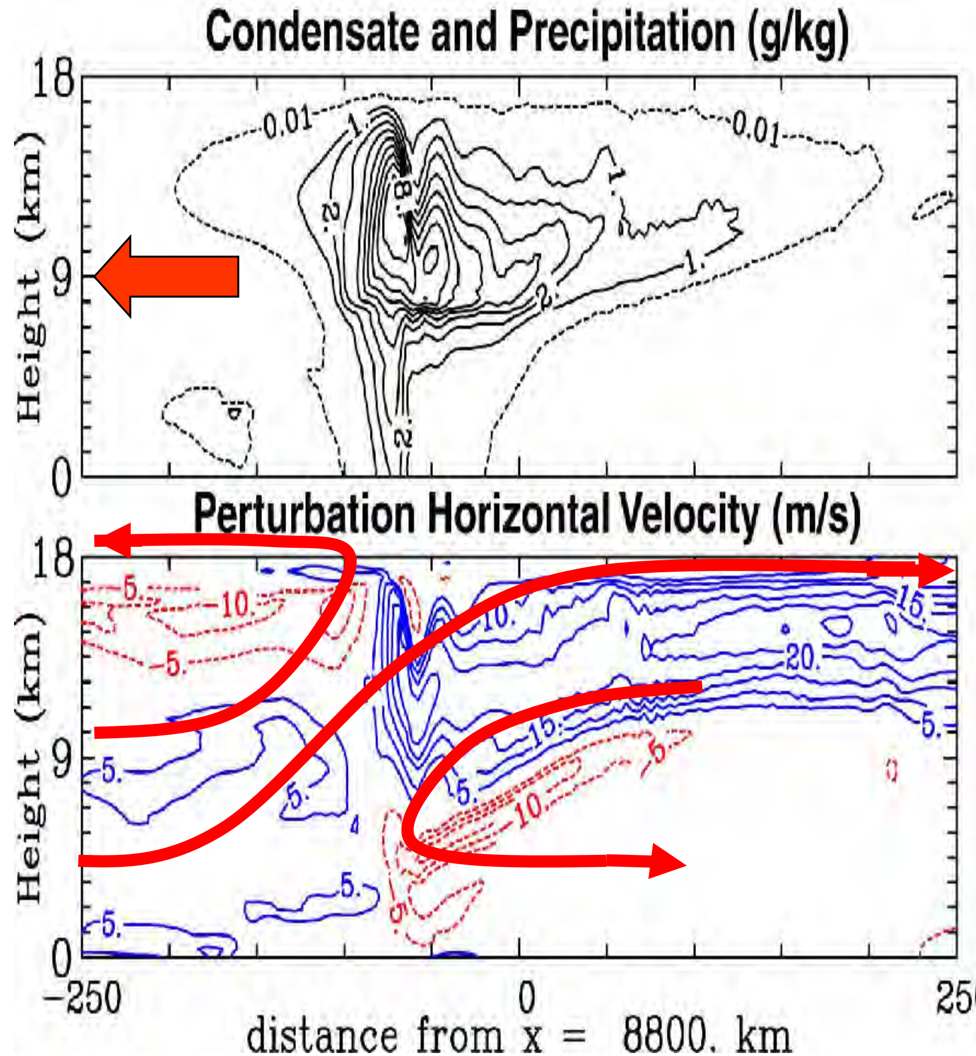
A: correct prediction B: over-prediction C: under-prediction D: correct no-convection



A: correct prediction B: over-prediction C: under-prediction
D: correct no-convection



1-D Cloud Model



Grabowski and Moncrieff (2001)

Mathematical Representation of Cloud Model

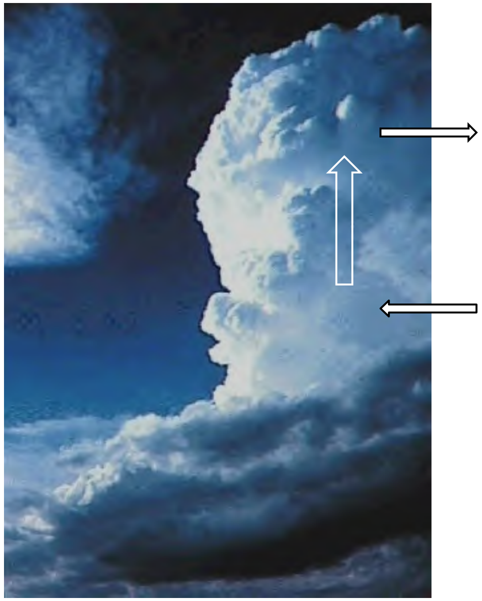
mass
$$\frac{\partial M_u}{\rho \partial z} = \varepsilon_u - \delta_u$$

heat
$$\frac{\partial M_u s_u}{\rho \partial z} = \varepsilon_u \overline{s} - \delta_u \hat{s}_u + L(c-e)$$

vapor
$$\frac{\partial M_u q_u}{\rho \partial z} = \varepsilon_u \overline{q} - \delta_u \hat{q}_u - (c-e)$$

liquid/ice
$$\frac{\partial M_u l}{\rho \partial z} = -\delta_u l + (c-e) - R_r / \rho$$

microphysics



ε_u and δ_u are the mass entrainment and detrainment. s_u and q_u are dry static energy and specific humidity in the updrafts.

$$q_u = q_s(s_u)$$

$$\hat{s}_u = \overline{s}$$

$$\hat{q}_u = q_s(\overline{s})$$

$$R_r = c_0 M_u l$$

$$q_u \approx q^*(s_u) = q^*[\overline{s} + (s_u - \overline{s})] = \overline{q}^* + \frac{1}{c_p}\left(\frac{\partial \overline{q}^*}{\partial \overline{T}}\right)(s_u - \overline{s})$$

$$h_u = s_u + Lq_u = s_u + L\overline{q}^* + \gamma(s_u - \overline{s})$$

$$= (s_u - \overline{s}) + (\overline{s} + L\overline{q}^*) + \gamma(s_u - \overline{s})$$

where $$\gamma = \frac{L}{c_p}\left(\frac{\partial \overline{q}^*}{\partial \overline{T}}\right)$$

$$s_u - \overline{s} = \frac{1}{1+\gamma}(h_u - \overline{h}^*)$$

$$q_u - \overline{q}^* = \frac{\gamma}{1+\gamma}(h_u - \overline{h}^*)$$

The updraft mass flux needs to be known and this is done through the specification of entrainment and detrainment.

$$\frac{\partial M_u}{\rho \partial z} = \varepsilon_u - \delta_u$$

Tiedtke (1989) partitioned the entrainment into organized inflow and turbulent entrainment, and the mass detrainment into organized outflow and turbulent detrainment:

$$E_u = E_u^1 + E_u^2, \qquad D_u = D_u^1 + D_u^2,$$

where $E_u^1 = \varepsilon_u M_u$ and $D_u^1 = \delta_u M_u$ are turbulent entrainment and detrainment, respectively.

The fractional entrainment/detrainment rates ε_u and δ_u are set to depend on cloud type.

- For shallow convection in convectively suppressed conditions, $\varepsilon_u = \delta_u = 3\times10^{-4}$ m^{-1}.
- For penetrative and mid-level convection in the presence of large-scale convergence, $\varepsilon_u = \delta_u = 1\times10^{-4}$ m^{-1}.

The different values for shallow and deep convection are meant to mimic the fact that shallow convective clouds are smaller in size and thus are subject to more entrainment from the cloud boundaries, whereas deep convective clouds under disturbed conditions are large and are subject to less entrainment.

For organized entrainment, in the case of deep convection Tiedtke (1989) assumed it to be proportional to large-scale moisture convergence:

$$E_u^2 = -\frac{\bar{r}}{\bar{q}}\left(\bar{\mathbf{v}} \cdot \nabla \bar{q} + \bar{w}\frac{\partial \bar{q}}{\partial z}\right)$$

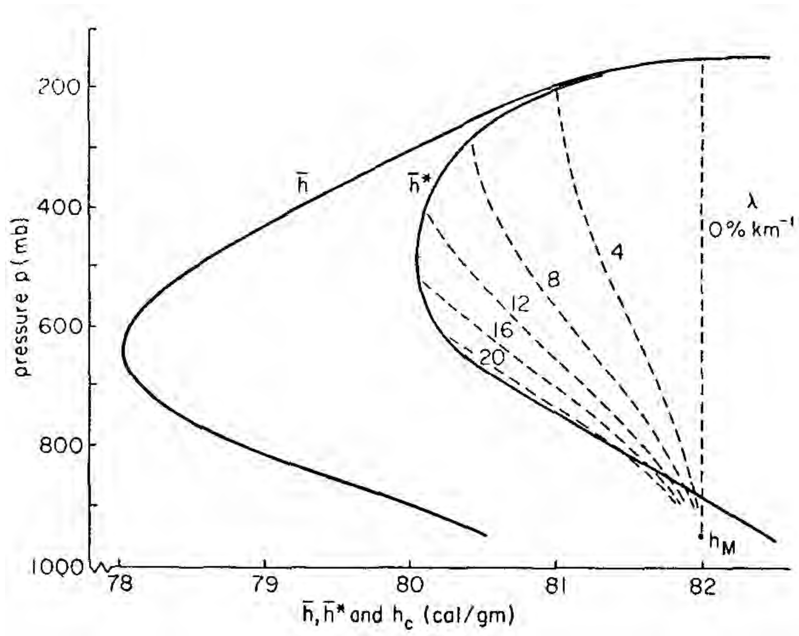
For shallow convection, no organized entrainment is present because these clouds often exist in regions of large-scale subsidence.

Organized detrainment for deep convection is assumed to occur only at the highest cloud layer, where all convective mass flux is detrained.

For shallow convection, organized detrainment is allowed to occur at the top most two cloud layers, with 70% of the mass flux detrained at the zero-buoyancy level and 30% detrained above it to represent overshooting into the inversion layer.

In ensemble plume models, only turbulent entrainment is considered.

To illustrate how the bulk mass flux is specified in this case, we follow the approach of Zhang and McFarlane (1995), which makes simplification of the Arakawa-Schubert (1974) spectral plume model.



For each cloud type with fractional entrainment rate λ, the variation of mass flux with height is given by:

$$\frac{\partial m_u(\lambda, z)}{\partial z} = \lambda m_u(\lambda, z)$$

Integrating from the cloud base z_b to z gives:

$$m_u(\lambda, z) = m_b(\lambda)\exp[\lambda(z - z_b)]$$

where $m_b(\lambda)$ is the updraft mass flux at the cloud base level for clouds spanning unit interval of fractional entrainment rates characterized by λ.

Integrating over all possible λ's that contribute to the mass flux at level z gives:

$$M_u(z) = \int_0^{\lambda_D(z)} m_b(\lambda)\exp[\lambda(z - z_b)]d\lambda$$

$\lambda_D(z)$ is the fractional entrainment rate of the updrafts that detrain at height z.

Assumptions:

1. Consider only a spectrum of clouds with fractional entrainment rate from 0 to a max value λ_0
2. cloud base mass flux for each cloud type is independent of the cloud type for the large range of deep clouds considered.

$$M_b = \int_0^{\lambda_0} m_b(\lambda)\, d\lambda$$

$$m_b(\lambda) = M_b / \lambda_0$$

$$M_u(z) = \frac{M_b}{\lambda_0 (z - z_b)} \left\{ \exp[\lambda_D(z)(z - z_b)] - 1 \right\}$$

To determine λ_D, consider a cloud type with entrainment rate λ. If the clouds are assumed to be in steady state, the moist static energy h_u of the updrafts satisfies

$$\frac{\partial h_u}{\partial z} + \lambda (h_u - h) = 0$$

At the cloud base, h_u is the same as the large-scale value h_b.
At the detrainment level, updraft moist static energy is equal to the saturated value of environmental value (the air temperature is the same as that of the environment, but is saturated).
Integrating from cloud base to the detrainment level gives:

$$h_b - h^*(z) = \lambda_D(z)\int_{z_b}^{z}\left[h_u(\lambda_D(z), z') - h_b\right]dz'$$

where h_u is the moist static energy in the updraft with fractional entrainment rate λ_D(z) and h^* is the saturation moist static energy.

The updraft moist static energy h_u can be expanded in a Taylor series in λ_D:

$$h_u(\lambda, z) = h_u(0, z) + \sum_n \frac{1}{n!} \frac{\partial^n h_u}{\partial \lambda^n}\Big|_{\lambda=0} \lambda^n$$

The first term $h_u(0,z)=h_b$. The derivatives wrt λ can be obtained by taking derivative of

$$\frac{\partial h_u}{\partial z} + \lambda (h_u - h) = 0$$

$$\frac{\partial}{\partial \lambda}\left(\frac{\partial h_u}{\partial z}\right) + \lambda \frac{\partial h_u}{\partial \lambda} + (h_u - h) = 0$$

note h is independent of λ

Integrating from z_b to z:

$$\frac{\partial h_u(z)}{\partial \lambda} - \frac{\partial h_u(z_b)}{\partial \lambda} + \lambda \int_{z_b}^{z} \frac{\partial h_u}{\partial \lambda} dz + \int_{z_b}^{z} (h_u - h)\, dz = 0$$

$$\left.\frac{\partial h_u(z)}{\partial \lambda}\right|_{\lambda=0} = -\int_{z_b}^{z} (h_b - h)\, dz$$

In general

$$\left.\frac{\partial^n h_u}{\partial \lambda^n}\right|_{\lambda=0} = (-1)^n n! I_n$$

$$I_n = \int_{z_b}^{z} \ldots (h_b - h) dz^{(n)}$$

$$h_b - h^*(z) = \lambda_D(z)\int_{z_b}^{z}\left[(h_b - h) + \sum_n (-1)^n I_n \lambda_D^n\right]dz' = I_1\lambda_D + \sum_n (-1)^n I_{n+1}\lambda_D^{n+1}$$

$$= I_1\lambda_D - I_2\lambda_D^2 + I_3\lambda_D^3 - I_4\lambda_D^4$$

Use series reversion technique to solve for λ_D

$$y = a_1 x + a_2 x^2 + a_3 x^3 + \ldots.$$

The series expansion of the inverse series is given by

$$x = A_1 y + A_2 y^2 + A_3 y^3 + \ldots.$$

Substituting x into the y equation gives

$$y = a_1 A_1 y + (a_2 A_1^2 + a_1 A_2) y^2 + (a_3 A_1^3 + 2 a_2 A_1 A_2 + a_1 A_3) y^3 + \ldots$$

$$A_1 = \frac{1}{a_1} \qquad A_2 = -\frac{a_2}{a_1^3} \qquad A_3 = \frac{2a_2^2 - a_1 a_3}{a_1^5}$$

$$\lambda_D = \mu + \frac{I_2}{I_1}\mu^2 + \frac{(2I_2^2 - I_1 I_3)}{I_1^2}\mu^3 + \ldots\ldots$$

$$\mu = \frac{h_b - h^*}{I_1}$$

To determine the mass detrainment from the subensemble of updrafts with tops at *z*, the same procedure that was used in Arakawa and Schubert (1974) can be applied here, i.e.

$$d_u(z) = -m_u(\lambda_D(z), z)\frac{d\lambda_D(z)}{\rho\, dz} = -\frac{M_b}{\lambda_0}\exp[\lambda_D(z)(z - z_b)]\frac{d\lambda_D(z)}{\rho\, dz}$$

Downdrafts

$$\frac{\partial M_d}{\rho \partial z} = -\delta_d + \varepsilon_d$$

$$\frac{\partial M_d s_d}{\rho \partial z} = -\delta_d \hat{s}_d + \varepsilon_d \overline{s}$$

$$\frac{\partial M_d q_d}{\rho \partial z} = -\delta_d \hat{q}_d + \varepsilon_d \overline{q}$$

Spectral Cloud Model

In spectral cloud model, each type of cloud mass flux and its corresponding thermodynamic properties are determined separately.

$$\frac{\partial \eta(z,\lambda)}{\partial z} = \lambda\,\eta(z,\lambda)$$

$$\eta(z,\lambda) = \exp\{\lambda(z - z_b)\},\quad z_b \le z \le z_D(\lambda)$$

$$\frac{\partial \eta(z,\lambda)h_c(z,\lambda)}{\partial z} = \frac{\partial \eta(z,\lambda)}{\partial z}\,\bar{h}(z)$$

$$\frac{\partial \eta(z,\lambda)[q_c(z,\lambda) + l(z,\lambda)]}{\partial z} = \frac{\partial \eta(z,\lambda)}{\partial z}\,\bar{q}(z) - \eta(z,\lambda)\,r(z,\lambda)$$

Vertical integration of the moist static energy equation gives:

$$h_c(z,\lambda) = \frac{1}{\eta(z,\lambda)}\left[h_c(z_b,\lambda) + \lambda \int_{z_b}^{z} \eta(z',\lambda)\,\bar{h}(z',\lambda)\,dz' \right]$$

In the above, cloud mass flux for each cloud type is normalized by its value at the cloud base level. The cloud base mass flux variation with λ is determined by the closure, for which the cloud work function is introduced. Cloud work function is the vertical integral of the buoyancy of a cloud parcel lifted adiabatically, weighted by the normalized cloud mass flux. This will be presented in the next lecture.

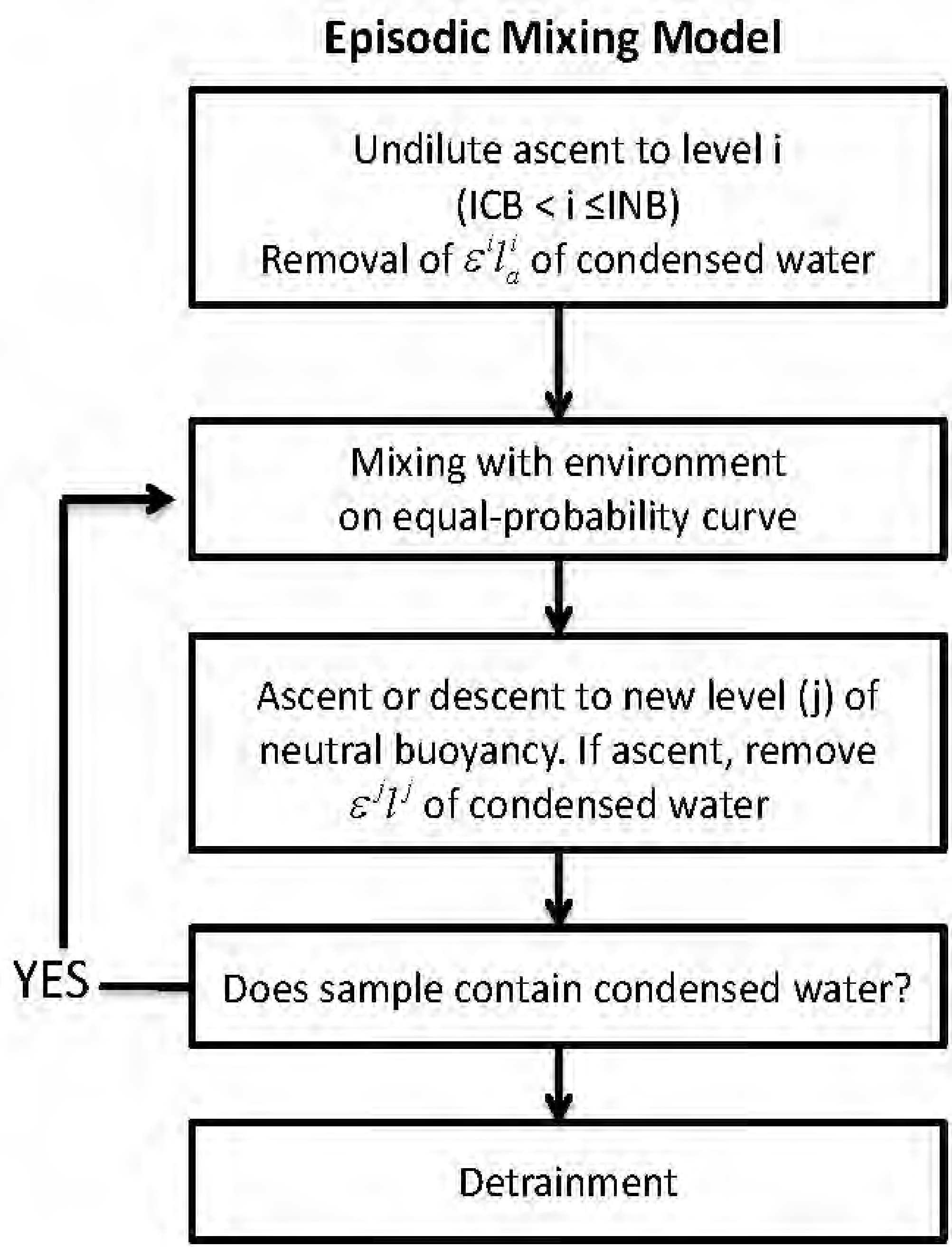
Episodic mixing model

The entraining plume model assumes that subcloud-layer air mixes thoroughly and continually with the environmental air as it rises in updrafts.

Raymond and Blyth (1986) introduced a stochastic mixing model, which assumes that an individual subcloud-layer air parcel undergoes only one mixing event on its way to the neutral buoyancy level where it detrains.

Based on this episodic mixing concept, Emanuel (1991) used a stochastic mixing model to represent convective updrafts.

- Each cloud of scale O(1 km) consists of sub-cloud scale O(100m) updrafts. An updraft rising from the cloud base to a level has an equal probability of mixing with air from all levels during its ascent.
- The mixed air then undergoes a further ascent or a descent to its neutral buoyancy level for detrainment, depending on the buoyancy of the mixture.
- In this buoyancy-sorting approach, cloud mass flux profile is determined by the vertical gradient of buoyancy (Emanuel and Zivkovic-Rothman 1999).


Episodic Mixing Model
Undilute ascent to level i
(ICB < i ≤INB)
Removal of $\varepsilon^i l_a^i$ of condensed water
Mixing with environment
on equal-probability curve
Ascent or descent to new level (j) of
neutral buoyancy. If ascent, remove
$\varepsilon^j l^j$ of condensed water
YES
Does sample contain condensed water?
Detrainment

Closures

- Closure is an empirical relationship between convection and model-resolved scale fields
- It determines the amount of convection given the large-scale atmospheric state
- There are as many closures as convective parameterization schemes.

Moisture Convergence Closure

Moisture convergence: The vertical integral of moisture convergence was used as a closure variable for convective parameterization by *Kuo* [1974], *Bougeault* [1985] and *Tiedtke* [1989]. It is defined as

$$M_c = \frac{1}{g}\int_0^{Ps}\left(u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} + w\frac{\partial q}{\partial z}\right)dp$$

where u, v and w are the zonal, meridional and vertical components of wind, q is the specific humidity, g is the acceleration due to gravity and P_s is the lowest model pressure level.

TKE-based Closure

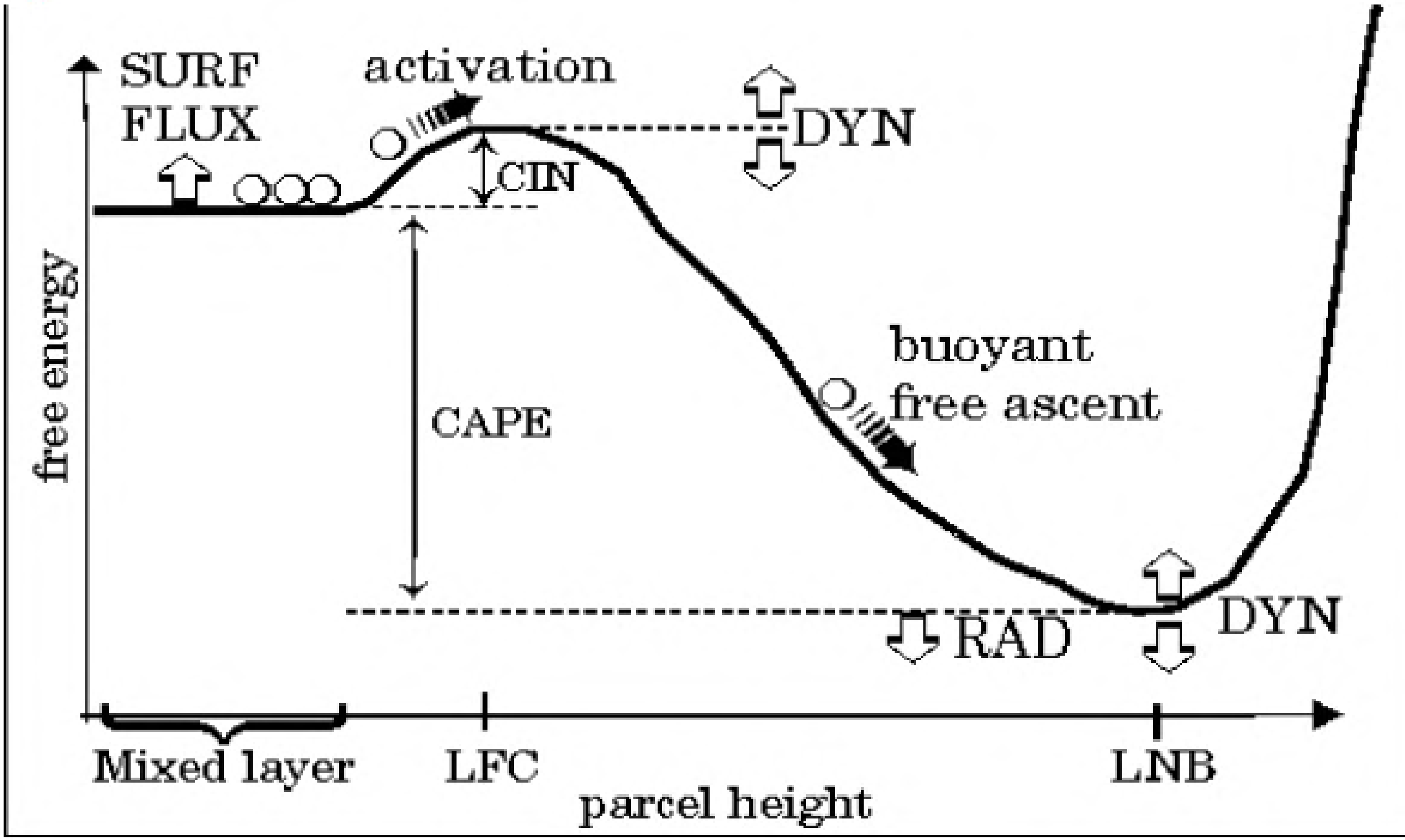
$$M_c \propto \sqrt{TKE}\,\exp(-CIN/TKE)$$

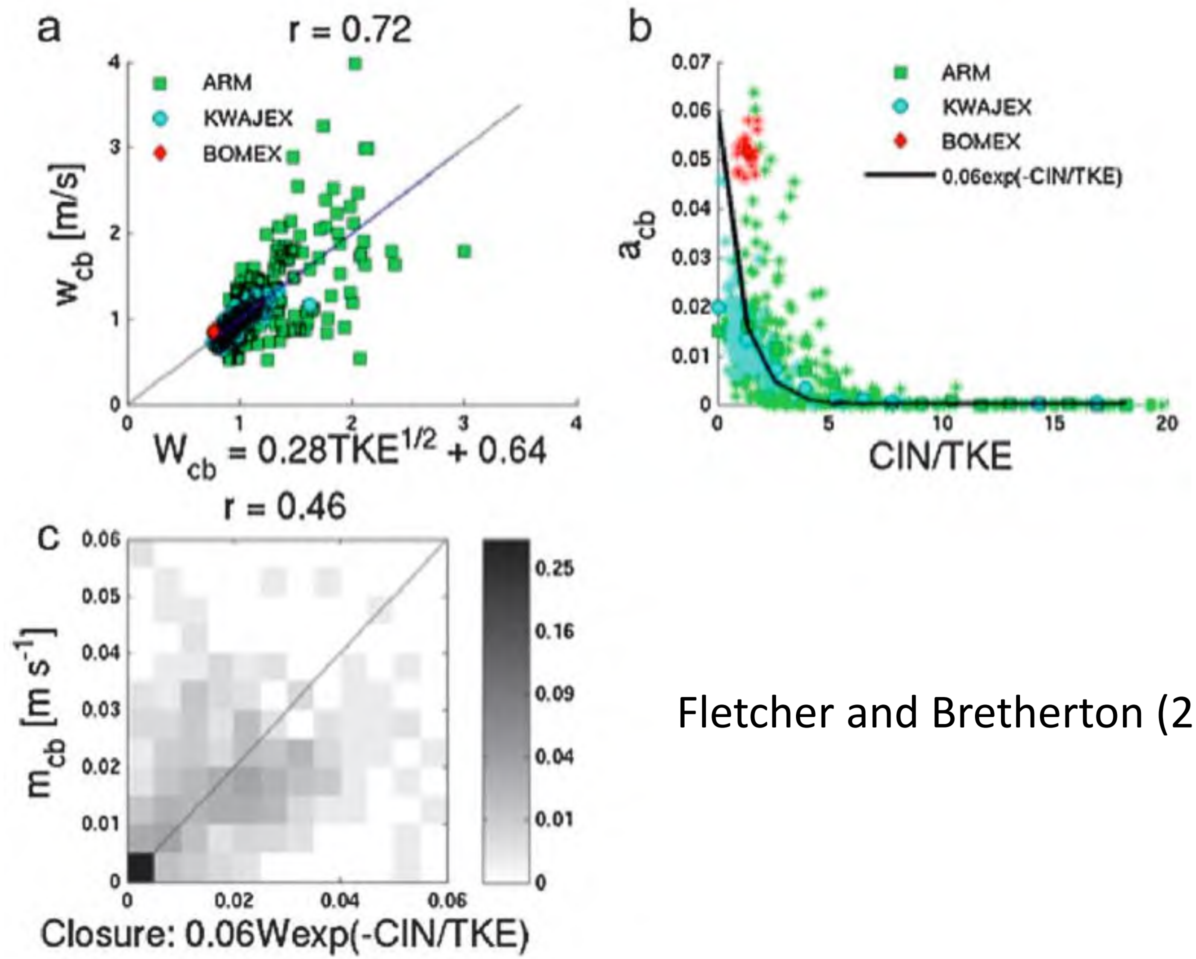
This is a closure proposed by activation control hypothesis and is mainly used in idealized models and convection schemes that extend the shallow convection scheme to deep convection schemes [*Mapes* 1997, *Fletcher and Bretherton* 2010, *Hohenegger and Bretherton* 2011]. CIN is defined as the vertical integral of buoyancy between the parcel source layer and the LFC.

$$CIN = -\int^{P_s} R_d \left(T_{vp} - T_{ve}\right) d\ln p$$

$$TKE = \frac{1}{\Delta p}\int^{P_s} \frac{1}{2}\left(u'^2 + v'^2 + w'^2\right) dp$$

where u' , v' and w' are the zonal, merdional and vertical wind anomalies from their respective domain mean. Δp is the thickness of the sub-cloud layer.


SURF
FLUX
activation
DYN
CIN
CAPE
buoyant
free ascent
RAD
DYN
free energy
Mixed layer
LFC
LNB
parcel height



Fletcher and Bretherton (2010)

CAPE Closure

$$\frac{dCAPE}{dt} = \left(\frac{dCAPE}{dt}\right)_{cu} + \left(\frac{dCAPE}{dt}\right)_{LS} \approx 0$$

Recall that

$$\frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\bar{\mathbf{v}}\,\bar{s}) + \frac{\partial(\bar{\rho}\,\bar{w}\,\bar{s})}{\bar{\rho}\,\partial z} = \bar{Q}_R + L(c - e) + \frac{\partial}{\bar{\rho}\,\partial z}[M_u(s_u - \bar{s}) + M_d(s_d - \bar{s})]$$

CAPE change due to convection can be written as

$$\left(\frac{\partial CAPE}{\partial t}\right)_{cu} = -M_b K \ ,$$

where K is the CAPE consumption rate by convection per unit cloud base updraft mass flux, and is determined by the large-scale thermodynamic profiles and the cloud model

The closure condition is that the CAPE is removed at an exponential rate by convection with a characteristic adjustment time scale τ.

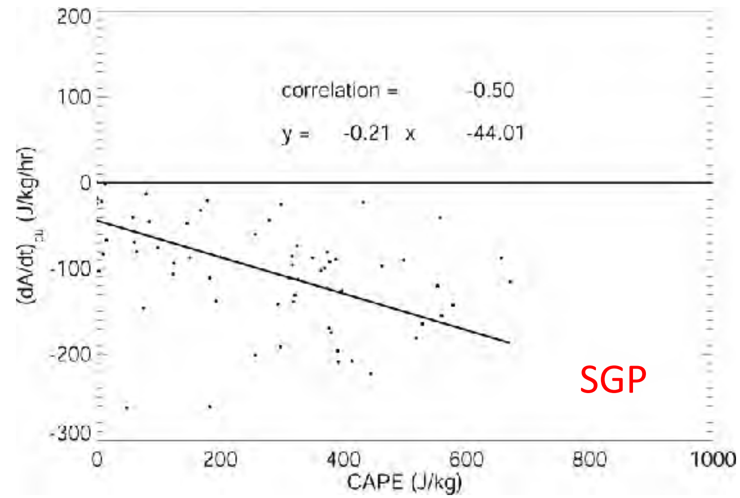
$$\left(\frac{\partial CAPE}{\partial t}\right)_{cu} = -\frac{CAPE}{\tau} ,$$

Thus

$$M_b = \frac{\mathrm{CAPE}}{\tau K} ,$$

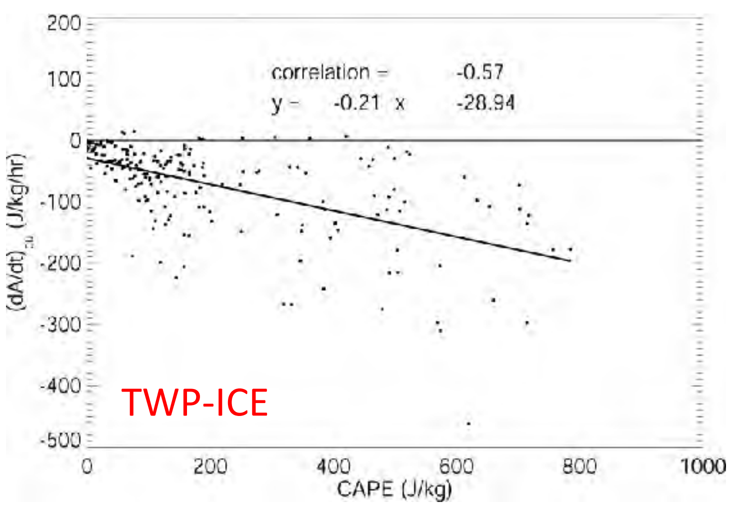
where τ is typically a few hours. This type of closure has also been used in the ECMWF Integrated Forecast System (Gregory et al. 2000) and the Hadley Centre climate model HadAM3 (Pope et al. 2000).

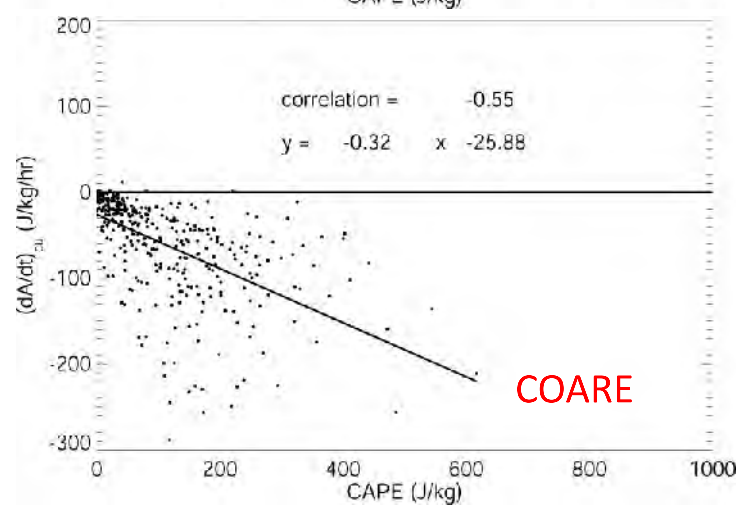
$$\left(\frac{\partial CAPE}{\partial t}\right)_{cu} = \frac{CAPE - CAPE_0}{\tau}$$



Convective closure:

$$M_c = \frac{CAPE - CAPE_0}{\tau F}$$

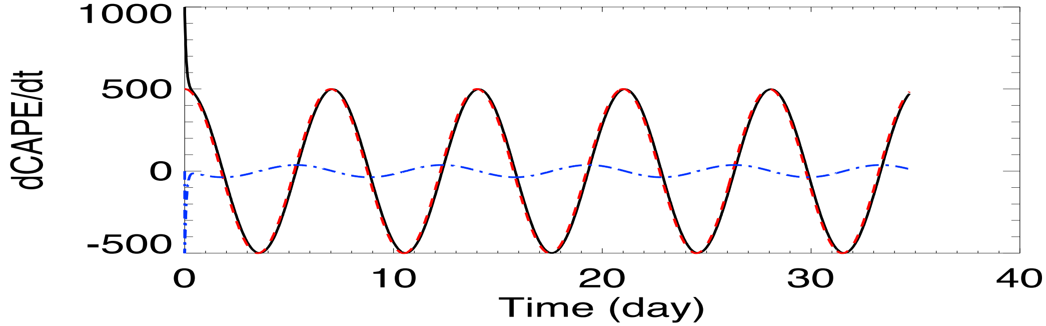




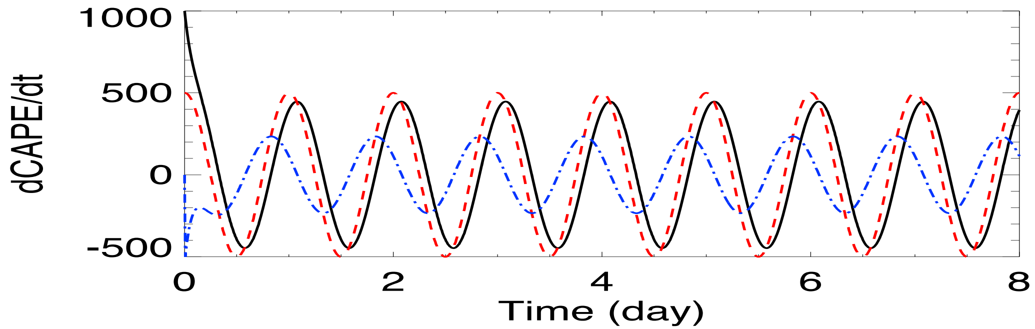
To see if use of $\left(\frac{\partial CAPE}{\partial t}\right)_{cu} = -\frac{CAPE}{\tau}$ satisfies $\frac{dCAPE}{dt} = \left(\frac{dCAPE}{dt}\right)_{cu} + \left(\frac{dCAPE}{dt}\right)_{LS} \approx 0$, we look at the following example.

$$\left(\frac{\partial CAPE}{\partial t}\right)_{LS} = F_0 \cos(2\pi t / T) ,$$

with F_0=500 J/kg/hr, T=7 days to emulate synoptic systems, and tau=2 hrs.



Red: $dCAPE_{ls}/dt$
Black: $dCAPE_{cu}/dt$
Blue: $dCAPE/dt$
T=7 days



T=1 day

Indeed, $-\left(\frac{dCAPE}{dt}\right)_{cu} \approx \left(\frac{dCAPE}{dt}\right)_{LS}$, and $\frac{dCAPE}{dt} \approx 0$ for T=7 days

However, if we choose T=1 day to emulate the surface forcing from diurnal cycle, with all other fields the same, there is a clear time lag between convective change and large-scale change of CAPE, and $\frac{dCAPE}{dt} \approx 0$ is no longer a good approximation. This suggests that scale separation is important for this closure to be valid.

Example of convective stabilization

Earlier we showed that change of CAPE is given by

$$\frac{\partial CAPE}{\partial t} = (T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t} - \int_{init}^{LNB}\left(\frac{g}{c_p}\frac{\dot{Q}}{T} - \frac{g}{q}(\mathbf{v}\cdot\nabla q + w\frac{\partial q}{\partial p})\right)dz$$

By analogy, the change of CAPE due to convection is

$$\frac{\partial CAPE}{\partial t}\bigg|_{convection} = (T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t}\bigg|_{convection} - \int_{init}^{LNB}\left(\frac{g}{c_p}\frac{\dot{Q}}{T} - \frac{g}{q}(\mathbf{v}\cdot\nabla q + w\frac{\partial q}{\partial p})\right)\bigg|_{convection} dz$$

Convective stabilization by downdrafts pouring into the PBL is

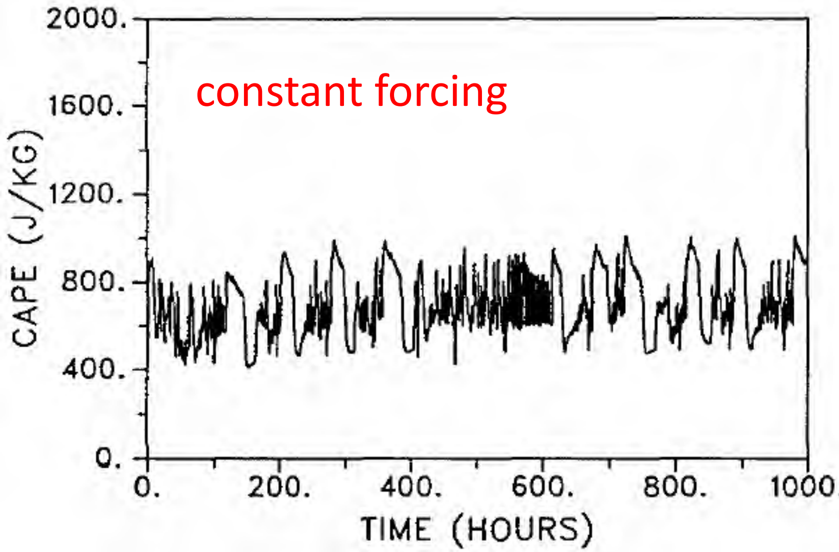
$$(T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t}\bigg|_{convection} \approx (T_{init} - T_{LNB})\frac{\beta M_c \triangle h}{T \triangle p}$$

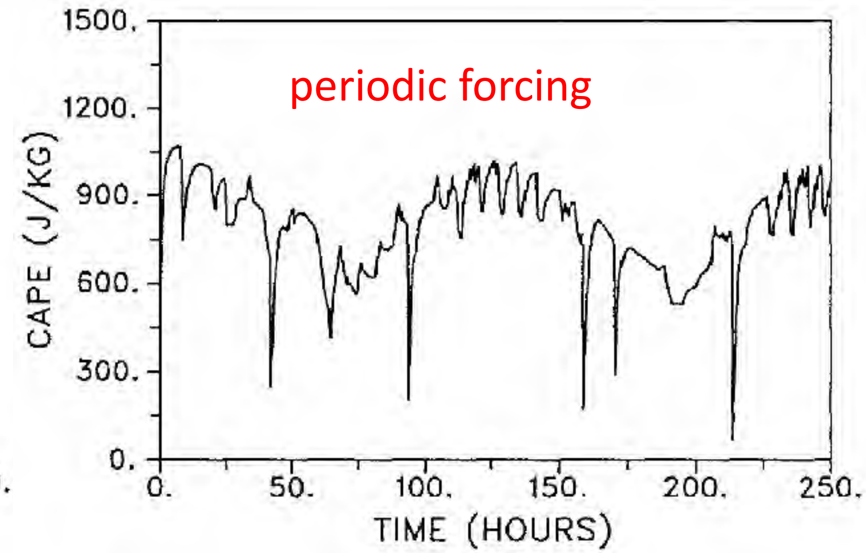
$$\sim 80K\frac{0.2\times 100mb/day(-30000J/kg)}{300K\times 50mb} = -3200J/kg/day$$

For tropical atmosphere, convective mass flux roughly balances the large-scale adiabatic upward motion. Thus, convective stabilization by compensating subsidence is about the same as the large-scale destabilization in magnitude

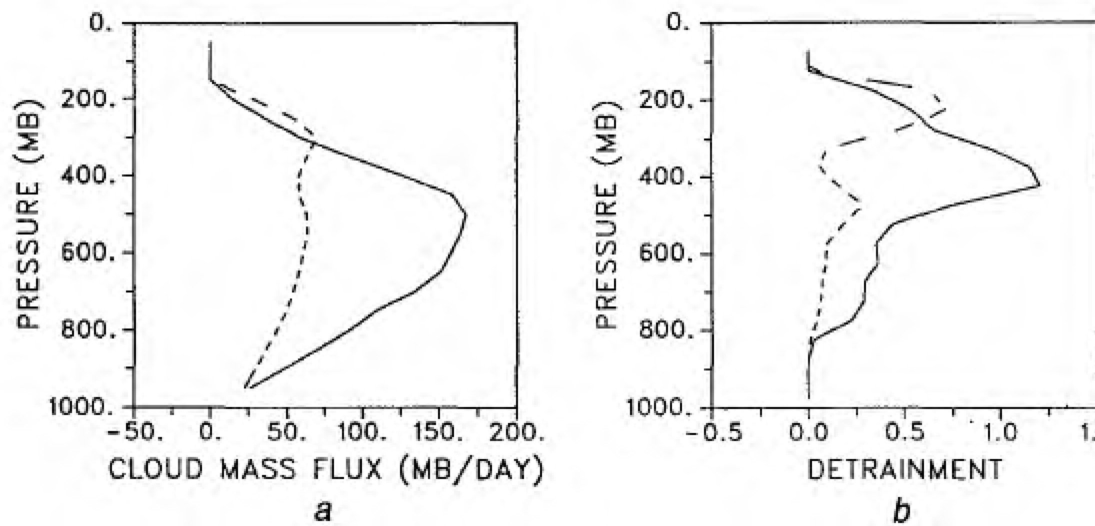
$$\frac{\partial CAPE}{\partial t} = \int_{init}^{LNB} \left(\frac{g}{\theta} \left(w \frac{\partial \theta}{\partial p} \right) \right) dz \approx \frac{10}{320} \frac{50 \sim 100mb}{day} \frac{50K}{900mb} 10^4 m \approx 850 \sim 1700 \quad \text{J/kg/day.}$$

Same as the effect of the large-scale forcing on CAPE, most of the convective stabilization of the atmosphere is achieved through PBL cooling and drying!





single column model test with idealized forcing (Zhang and McFarlane 1995)



Boundary Layer Quasi-equilibrium Closure

Raymond (1995) proposed a boundary layer quasi-equilibrium in which moist static energy (or moist entropy) generated by surface flux and depleted by downdrafts is in balance.

$$F_s = M_d \delta s_d$$

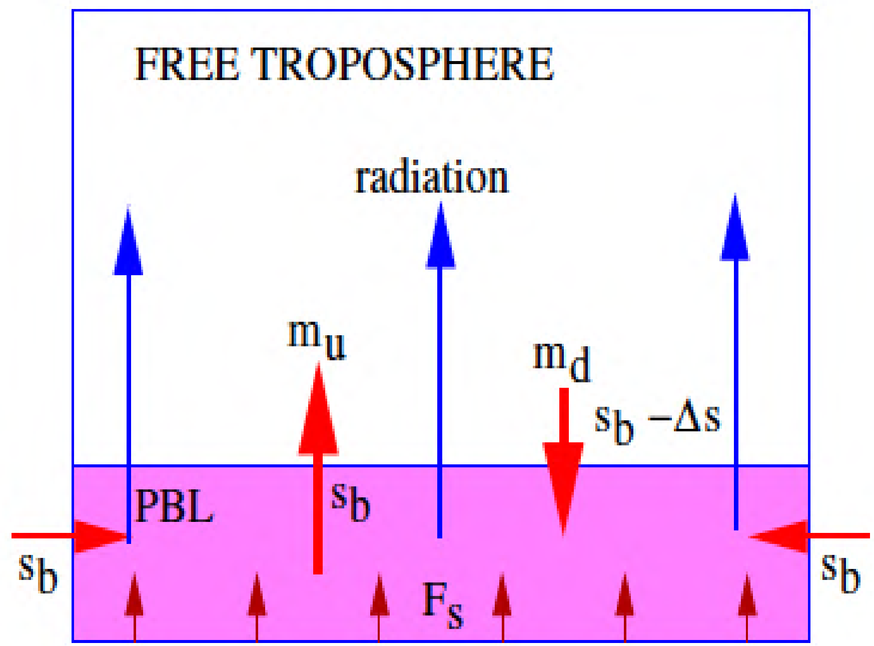
He further assumed that downdraft mass flux is proportional to updraft mass flux

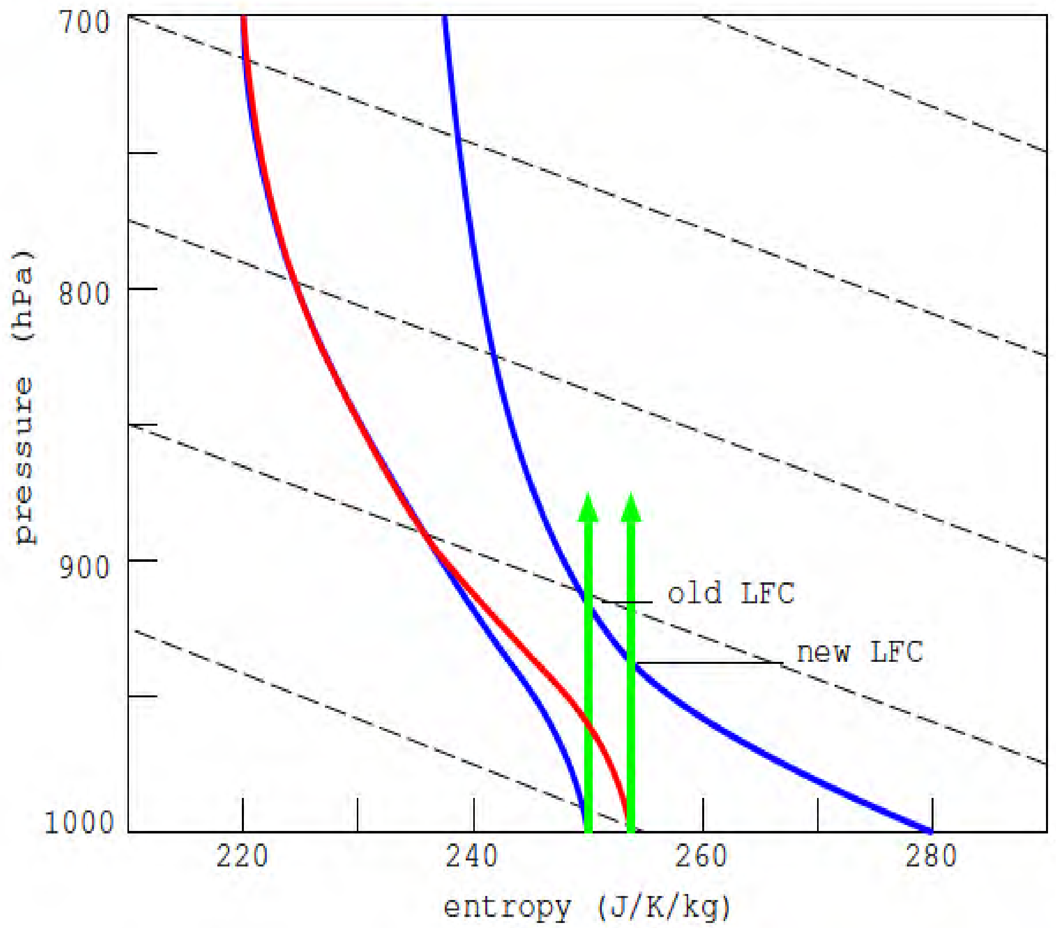
$$M_d = \alpha M_u$$

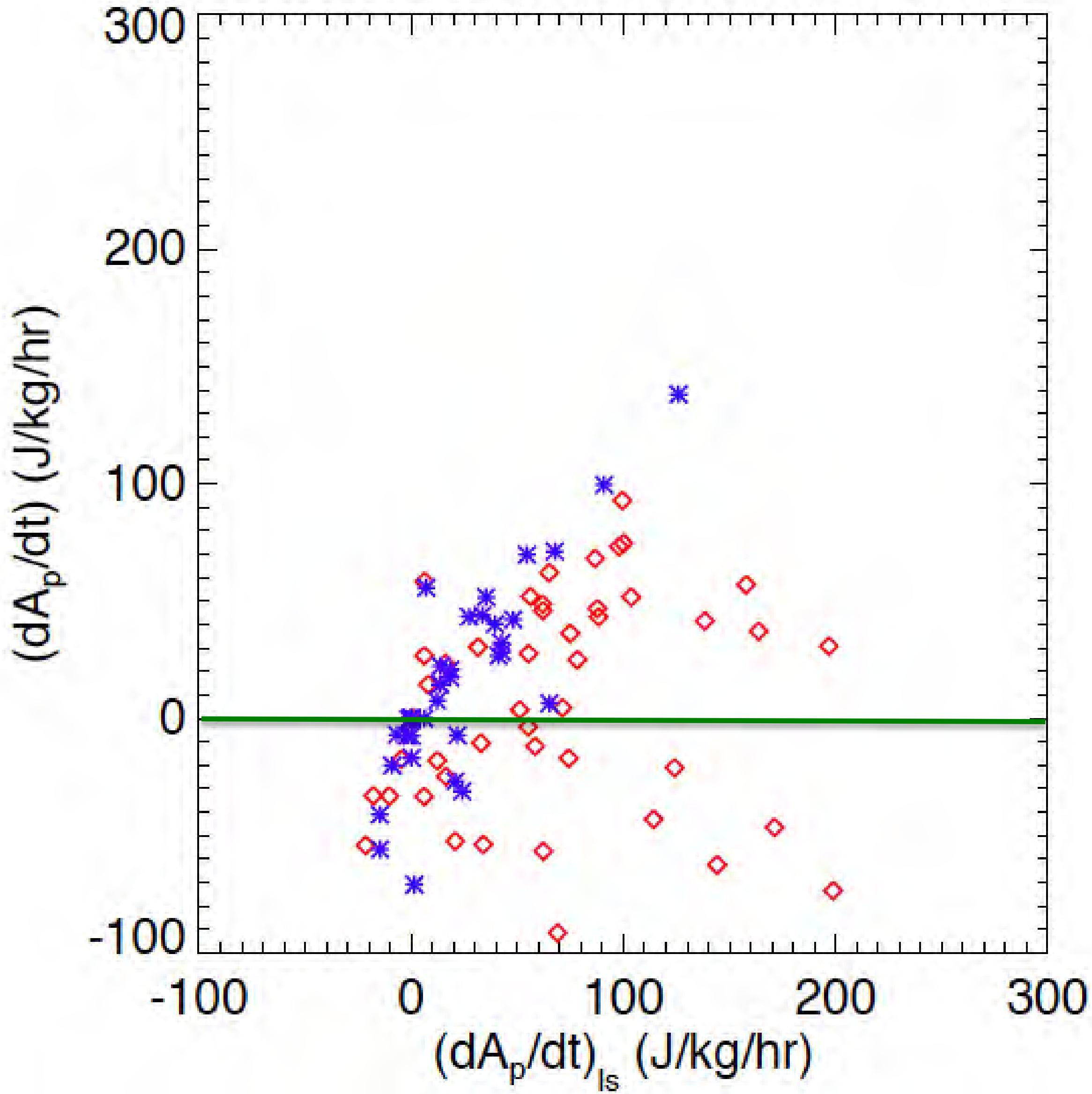
Combine these together we get

$$M_u = \frac{F_s}{\alpha \delta s_d}$$

$$\frac{dCAPE}{dt} = \left(\frac{dCAPE}{dt}\right)_{cu} + \left(\frac{dCAPE}{dt}\right)_{LS} \approx 0$$




700
800
900
1000
pressure (hPa)
220
240
260
280
entropy (J/K/kg)
old LFC
new LFC



300
200
100
0
-100
(dAp/dt) (J/kg/hr)
-100
0
100
200
300
(dAp/dt)ls (J/kg/hr)

Lower tropospheric quasi-equilibrium

Updraft mass flux is proportional to entropy difference between PBL and saturation value of the layer above

$$M_u = \gamma(s_{bl} - s_{lt}^*)$$

$$\frac{dM_u}{dt} = \gamma\left(\frac{ds_{bl}}{dt} - \frac{ds_{lt}^*}{dt}\right) = \gamma\left[\frac{F_s - \alpha M_u \delta s_d}{\rho H_b} - \frac{ds_{lt}^*}{dt}\right]$$

H_b is PBL thickness.

$$s = c_{pd}\ln T - R_d \ln p + \frac{Lr}{T} = c_{pd}\ln\theta + \frac{Lr}{T}, \text{ we get}$$

$$\frac{ds_{lt}^*}{dt} = \frac{ds_{lt}^*}{d\theta}\frac{d\theta}{dt} = \frac{c_{pd}}{\theta}\left[1 + \frac{L^2 r^*}{R_v c_{pd} T^2}\right]\frac{d\theta}{dt}$$

If there is a lifting, potential temperature change due to it is

$$\frac{d\theta}{dt} = -w_{lt}\frac{\partial\theta}{\partial z}, \text{ we have}$$

$$\frac{ds_{lt}^{*}}{dt} = -\chi w_{lt}$$

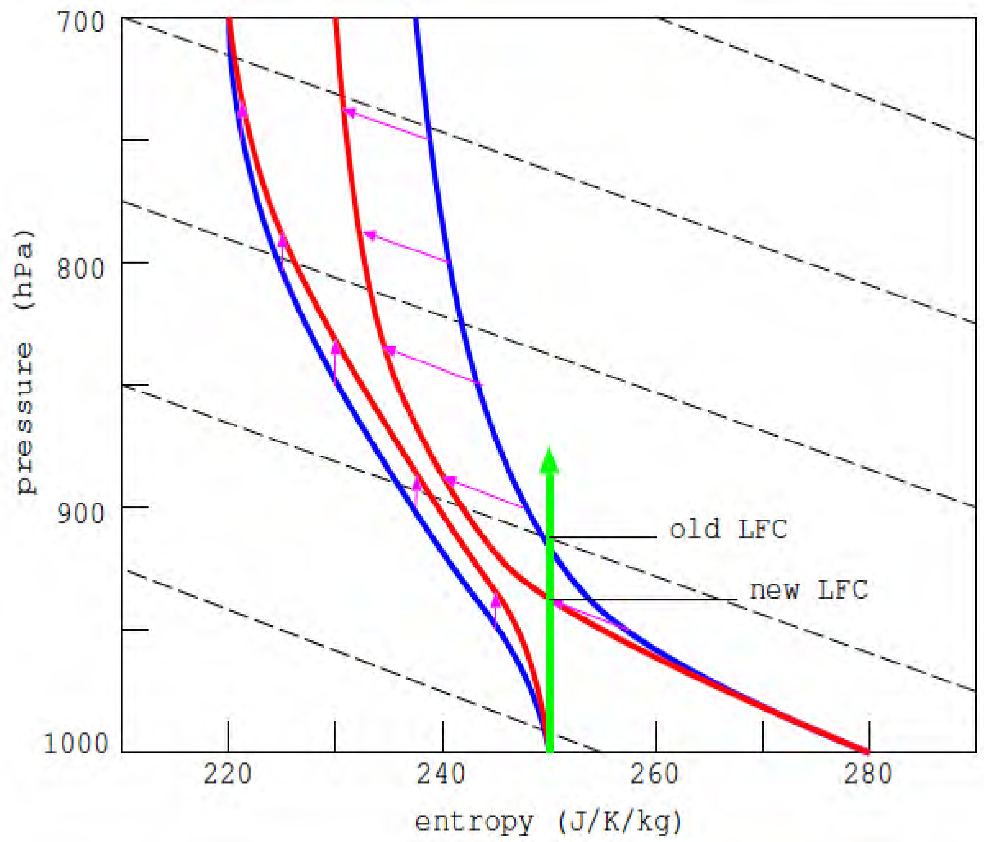
Thus,

$$\frac{dM_u}{dt} = \gamma\left[\frac{F_s - \alpha M_u \delta s_d}{\rho H_b} + \chi w_{th}\right]$$

If evolution of convective mass flux is slow and can be ignored,

$$M_u = \frac{F_s + \rho H_b \chi w_{lt}}{\alpha \delta s_d}$$

Lifting in the lower troposphere can increase convection



700
800
900
1000
pressure (hPa)
220
240
260
280
entropy (J/K/kg)
old LFC
new LFC

Arakawa-Schubert Convective Quasi-equilibrium

$$\frac{dA(\lambda)}{dt} = \frac{dA_c(\lambda)}{dt} + \frac{dA_{ls}(\lambda)}{dt} \approx 0$$

$dA_c(\lambda)/dt$ and $dA_{ls}(\lambda)/dt$ are the time rate of change of cloud work function due to convective and the large-scale processes, respectively.

$$A(\lambda) = \int_{p_t}^{p_b} R_d \eta(\lambda)(T_{vp}(\lambda) - T_{ve})\, d\ln p$$

$$= \int_{z_b}^{z_D(\lambda)} \frac{g}{c_p T} \eta(z,\lambda)(s_{vc}(z,\lambda) - \overline{s}_v(z))\, dz \text{ ,}$$

where p_b and p_t are the pressure at the parcel's initial level, i.e. the boundary layer, and the neutral buoyancy level, respectively, R_d is the gas constant of the dry air, T_{vp} and T_{ve} are the virtual temperature of the cloud parcel and its environment, respectively.

Convective stabilization is related to the grid-resolvable destabilization for each cloud type:

$$\frac{dA_c(\lambda)}{dt} = -\frac{dA_{ls}(\lambda)}{dt} \approx -\frac{A_{ls}^t(\lambda) - A^{t-\Delta t}(\lambda)}{\Delta t},$$

where $A_{ls}^t(\lambda)$ is the cloud work function at time t after the large-scale forcing is applied, $A^{t-\Delta t}$ is the observed cloud work function at $t - \Delta t$ (after convection), Δt is the time interval of the observations.

Lord (1982) estimated the observed cloud work function in the atmosphere using several different datasets in the tropics, and found that for a given cloud type they are in general invariant with time and can be replaced by a climatological value A_0. The values of A_0 for different l are given in Lord (1982). Thus, Eq. (39) can be written as:

$$\frac{dA_c(\lambda)}{dt} \approx -\frac{A_{ls}^t(\lambda) - A_0(\lambda)}{\Delta t}.$$

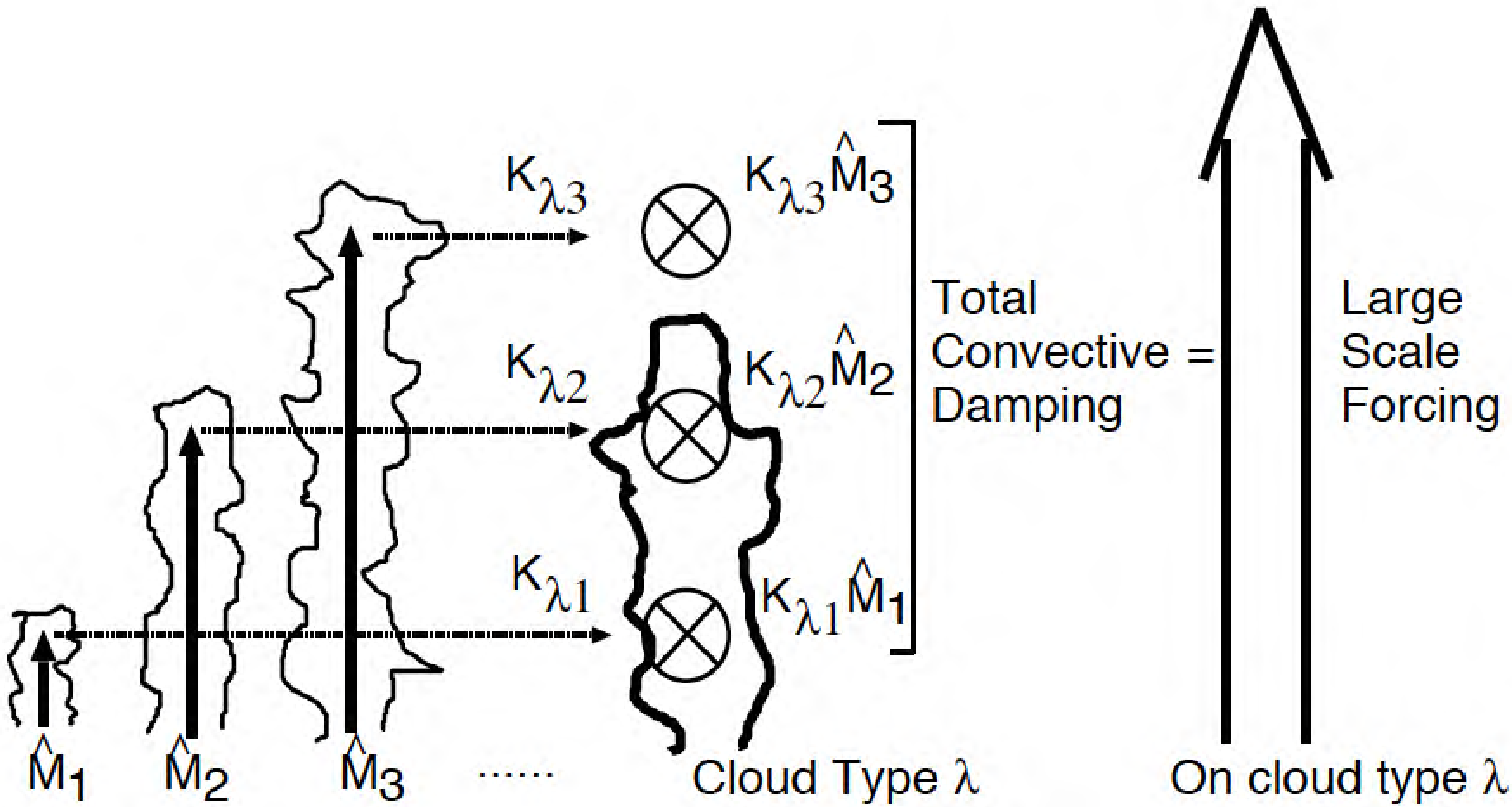
Arakawa and Schubert (1974) showed that the change in cloud work function by convection is proportional to the cloud base mass flux:

$$\frac{dA_c(\lambda)}{dt} = -F(\lambda)m_b(\lambda)$$

where $F(\lambda)$ can be computed from the large-scale conditions and the spectral cloud model. Thus, the closure equation becomes:

$$m_b(\lambda) = \frac{1}{F(\lambda)} \frac{A_{ls}^t(\lambda) - A_0(\lambda)}{\Delta t}.$$

For simplicity, we have not considered cloud-cloud interaction. In reality cloud base mass flux for one cloud type affects the cloud work function of another cloud type.


$K_{\lambda 3}$
$K_{\lambda 3}\hat{M}_3$
Total
Convective =
Damping
Large
Scale
Forcing
$K_{\lambda 2}$
$K_{\lambda 2}\hat{M}_2$
$K_{\lambda 1}$
$K_{\lambda 1}\hat{M}_1$
$\hat{M}_1$
$\hat{M}_2$
$\hat{M}_3$
......
Cloud Type λ
On cloud type λ

What does quasi-equilibrium imply on the tropical atmospheric thermodynamic structure?

$$\frac{\partial CAPE}{\partial t} = \int_{LNB}^{init} \left(\frac{\partial \alpha_p}{\partial t} - \frac{\partial \alpha}{\partial t}\right) dp = (T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t} - \int_{LNB}^{init} \frac{\partial \alpha}{\partial t} dp$$

But using hydrostatic relationship $\alpha = -\frac{\partial \phi}{\partial p}$, we have

$$\frac{\partial CAPE}{\partial t} = \int_{LNB}^{init} \left(\frac{\partial \alpha_p}{\partial t} - \frac{\partial \alpha}{\partial t}\right) dp = (T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t} + \int_{LNB}^{init} \frac{\partial}{\partial t}\frac{\partial \phi}{\partial p} dp \approx 0$$

$$\frac{\partial (\phi_{LNB} - \phi_i)}{\partial t} = (T_{init} - T_{LNB})\frac{\partial s_{init}}{\partial t}$$

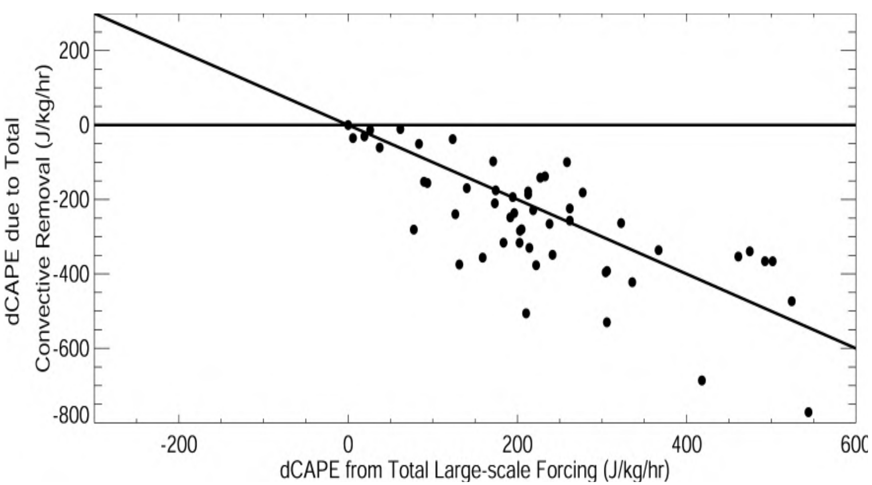
Time variation of the thickness of the tropical atmosphere is proportional to that of the PBL entropy.

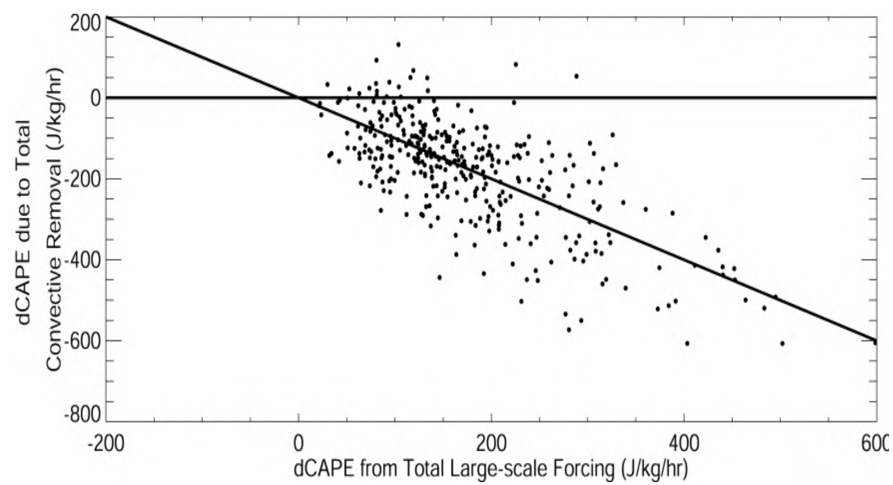
Free Tropospheric Quasi-Equilibriunm

$$\text{CAPE} \equiv A(0) = \int_{p_t}^{p_b} R_d (T_{vp} - T_{ve})\, d\ln p$$

$$\frac{dA}{dt} = \frac{dA_c}{dt} + \frac{dA_{ls}}{dt} \quad \text{or} \quad \frac{dA_c}{dt} = -\frac{dA_{ls}}{dt} + \frac{dA}{dt}$$

$$\left| dA/dt \right| << \left| dA_{ls}/dt \right|$$





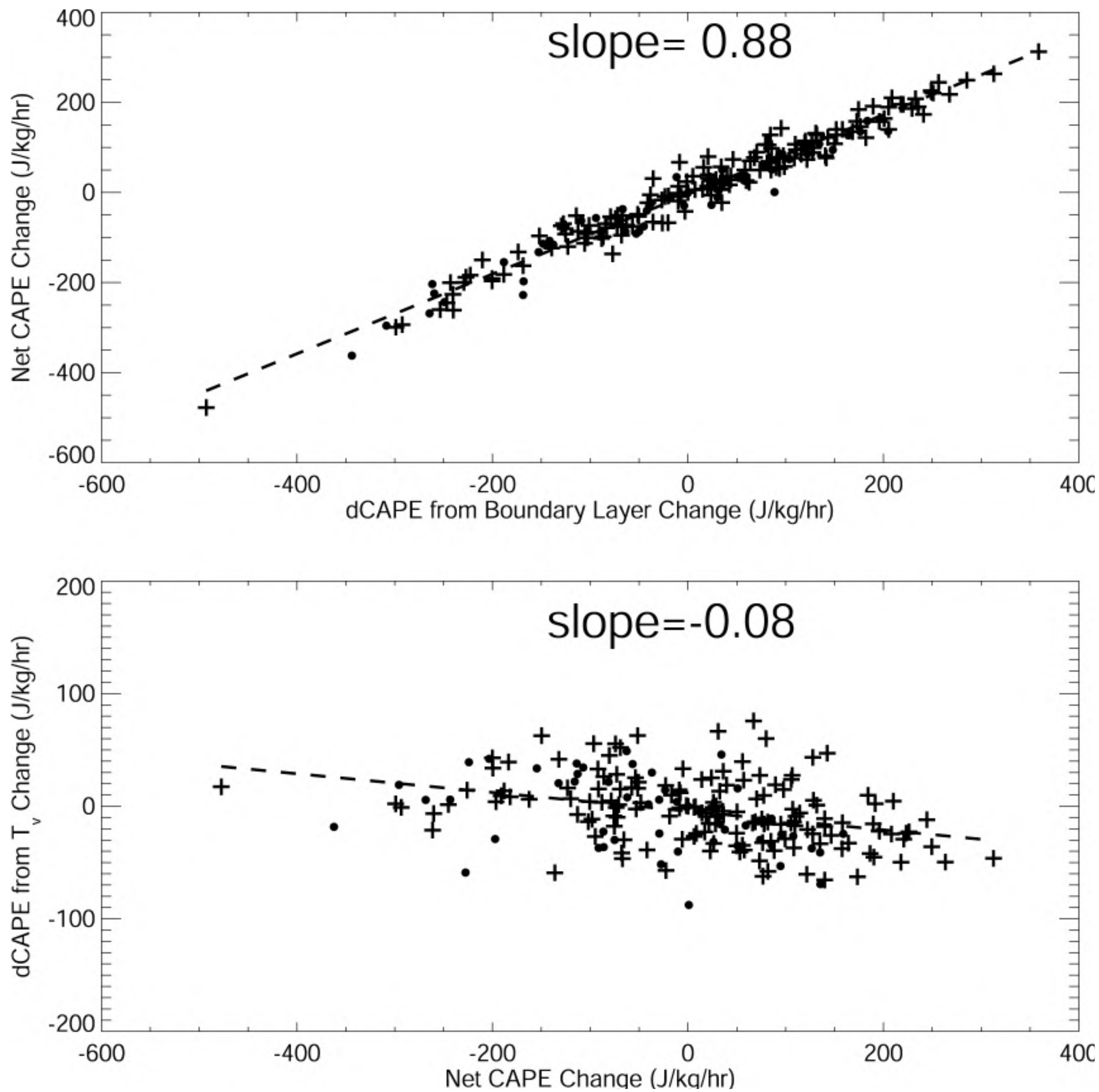
$$\frac{dA}{dt} = \frac{d}{dt}\left\{\int_{p_t}^{p_b} R_d (T_{vp} - T_{ve}) d \ln p\right\}$$

$$= \int_{p_t}^{p_b} R_d \left(\frac{dT_{vp}}{dt} - \frac{dT_{ve}}{dt} \right) d \ln p - R_d \left[T_{vp} - T_{ve} \right]_{p_t} \frac{dp_t}{dt}$$

$$\frac{dA}{dt} = \frac{dA^p}{dt} + \frac{dA^e}{dt}$$

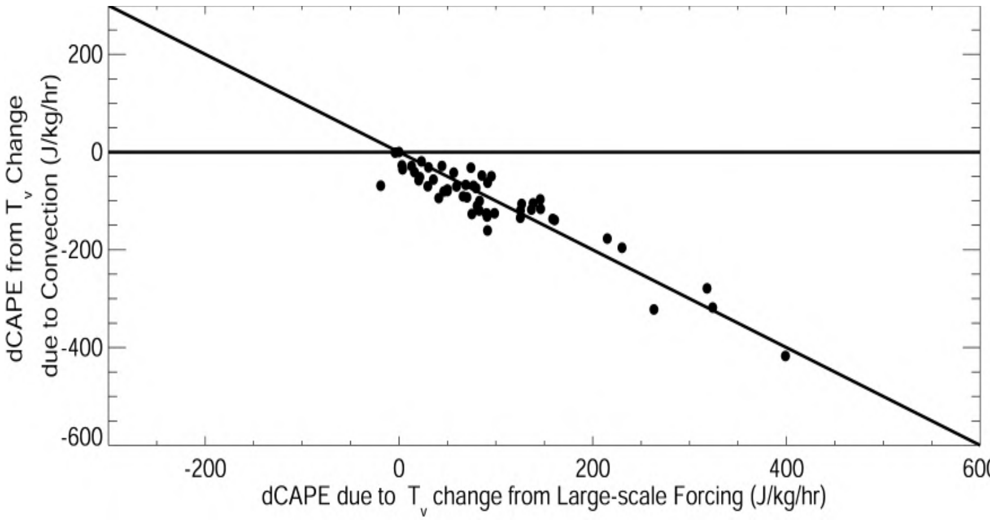
$$\frac{dA^p}{dt} = R_d \int_{p_t}^{p_b} \frac{dT_{vp}}{dt} d \ln p$$

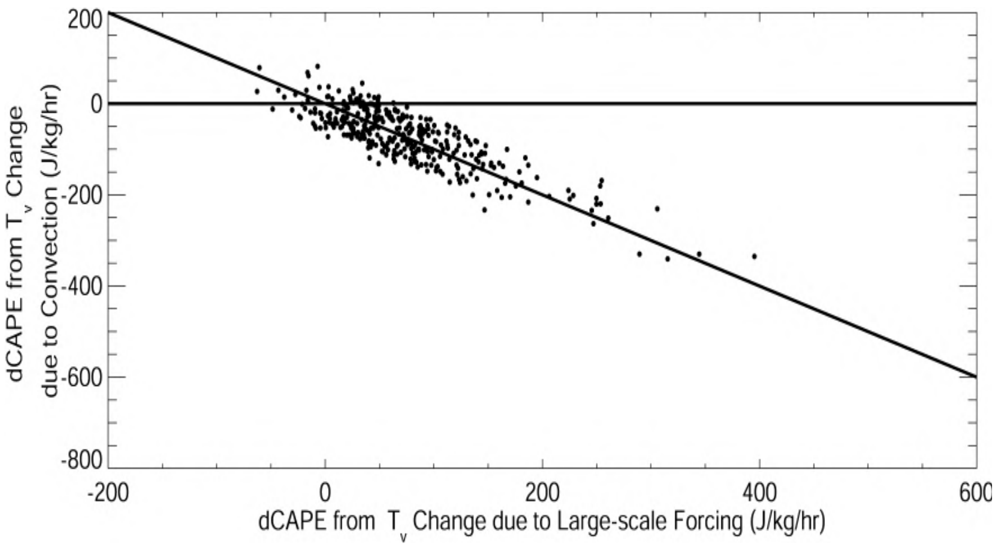
$$\frac{dA^e}{dt} = - R_d \int_{p_t}^{p_b} \frac{dT_{ve}}{dt} d \ln p$$

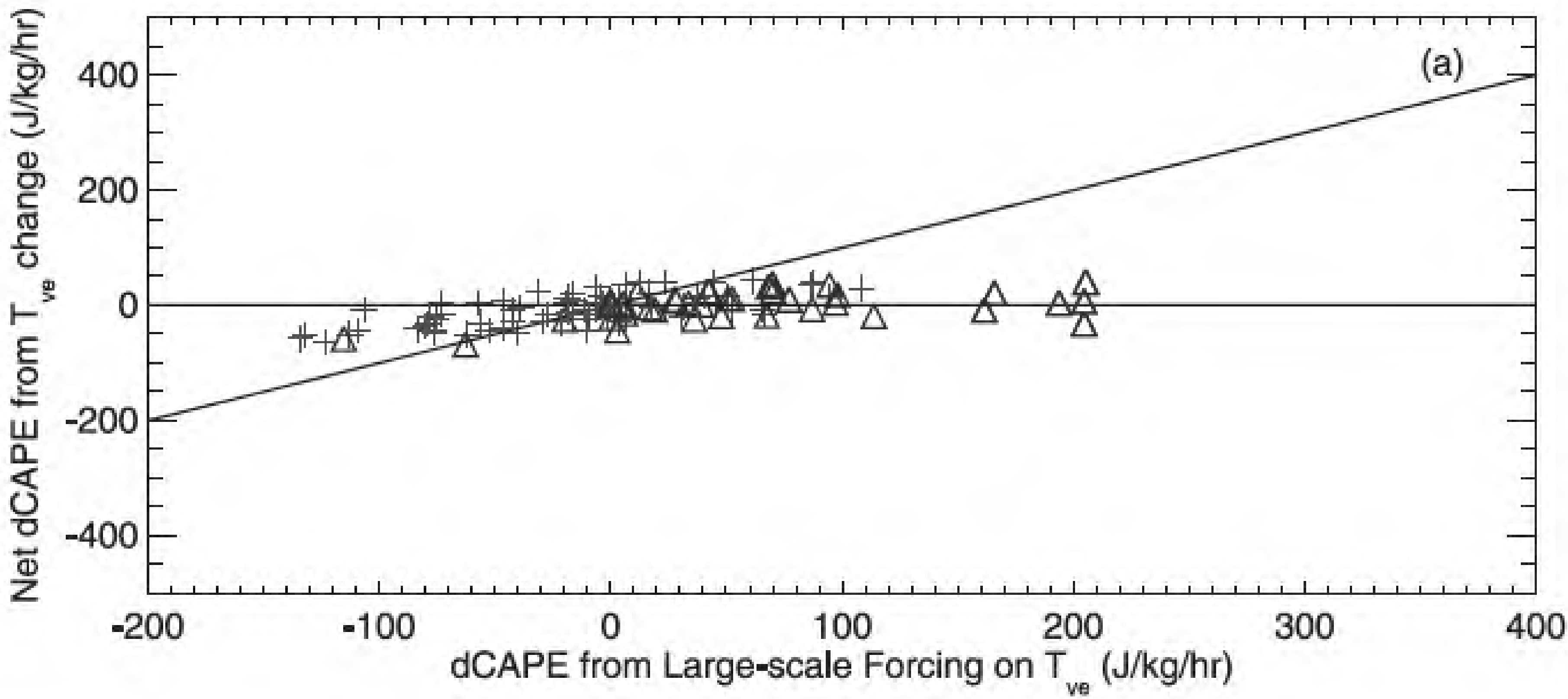


slope= 0.88
Net CAPE Change (J/kg/hr)
dCAPE from Boundary Layer Change (J/kg/hr)
400
200
0
-200
-400
-600
-600
-400
-200
0
200
400
slope=-0.08
dCAPE from T_v Change (J/kg/hr)
Net CAPE Change (J/kg/hr)
200
100
0
-100
-200
-600
-400
-200
0
200
400

$$\frac{dA^e}{dt} = \frac{dA^e_c}{dt} + \frac{dA^e_{ls}}{dt} \quad \text{or} \quad \frac{dA^e_c}{dt} = - \frac{dA^e_{ls}}{dt} + \frac{dA^e}{dt}$$







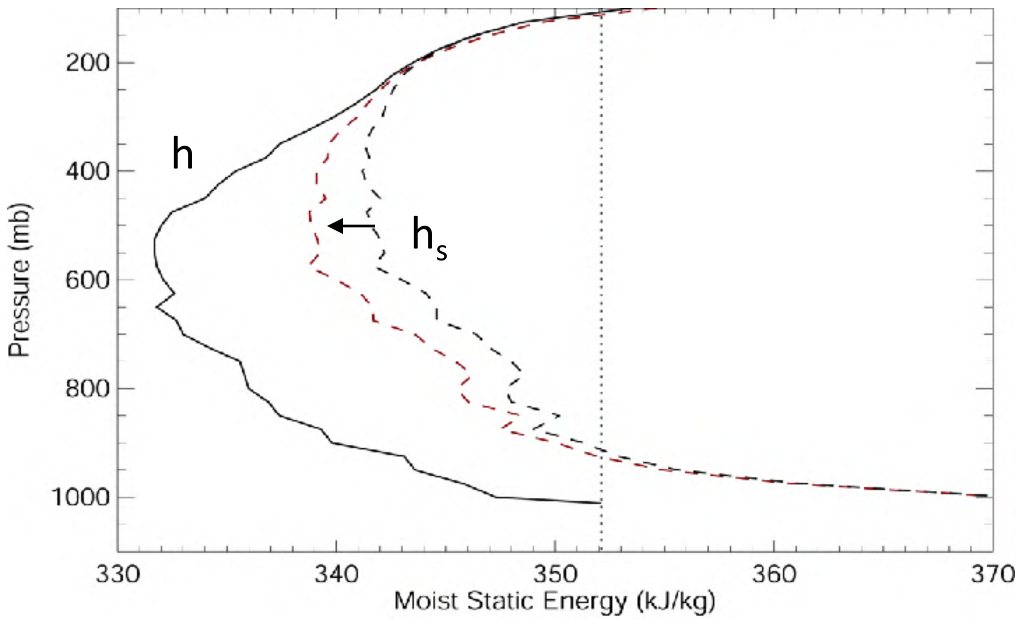
(a)
Net dCAPE from T_{ve} change (J/kg/hr)
400
200
0
-200
-400
-200
-100
0
100
200
300
400
dCAPE from Large-scale Forcing on T_{ve} (J/kg/hr)

$$\left(\frac{\partial T}{\partial t}\right)_{cu} = -M_b \eta \frac{\partial S}{\partial p}$$

$$\left(\frac{\partial q}{\partial t}\right)_{cu} = M_b \left[-\eta \frac{\partial q}{\partial p} + \delta(q_s - q)\right],$$

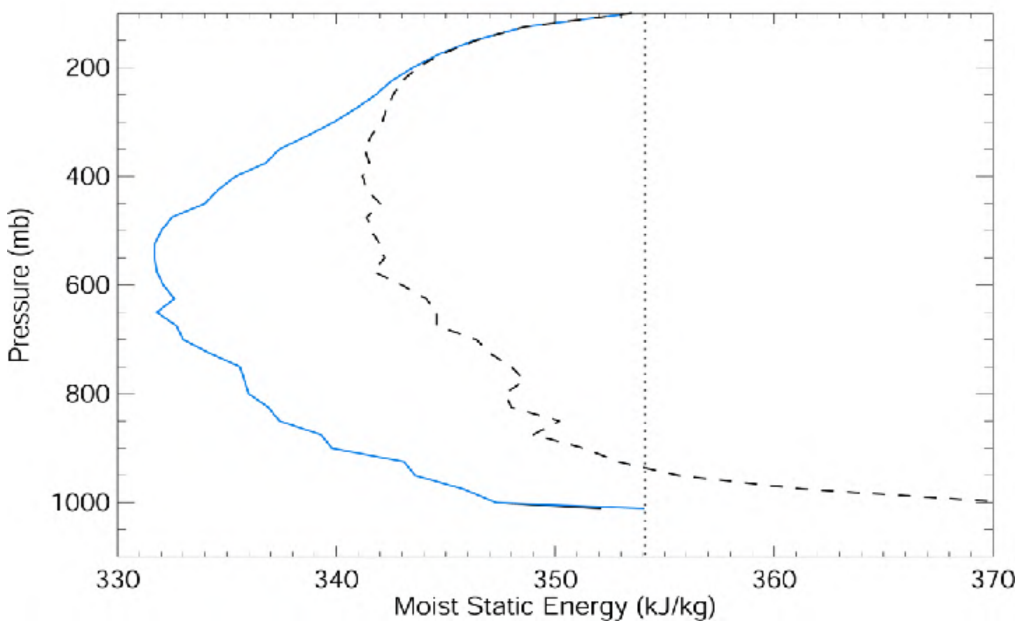
$$M_b = \frac{1}{k} \max\left\{-\int_{p_t}^{p_b} \left(\frac{\partial T_{ve}}{\partial t}\right)_{ls} d \ln p, 0\right\},$$

$$k = \int_{p_t}^{p_b} (1 + 0.608q) \left[-\eta \frac{\partial S}{\partial p}\right] + 0.608T \left[-\eta \frac{\partial q}{\partial p} + \delta(q_s - q)\right] d \ln p.$$

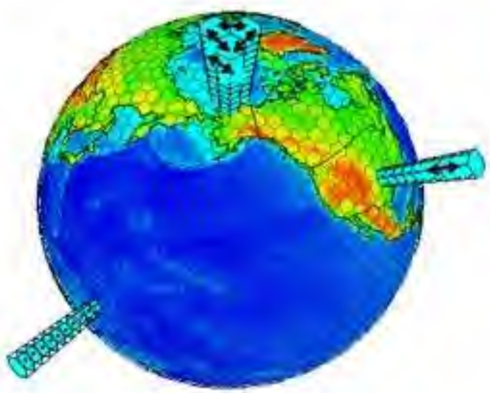


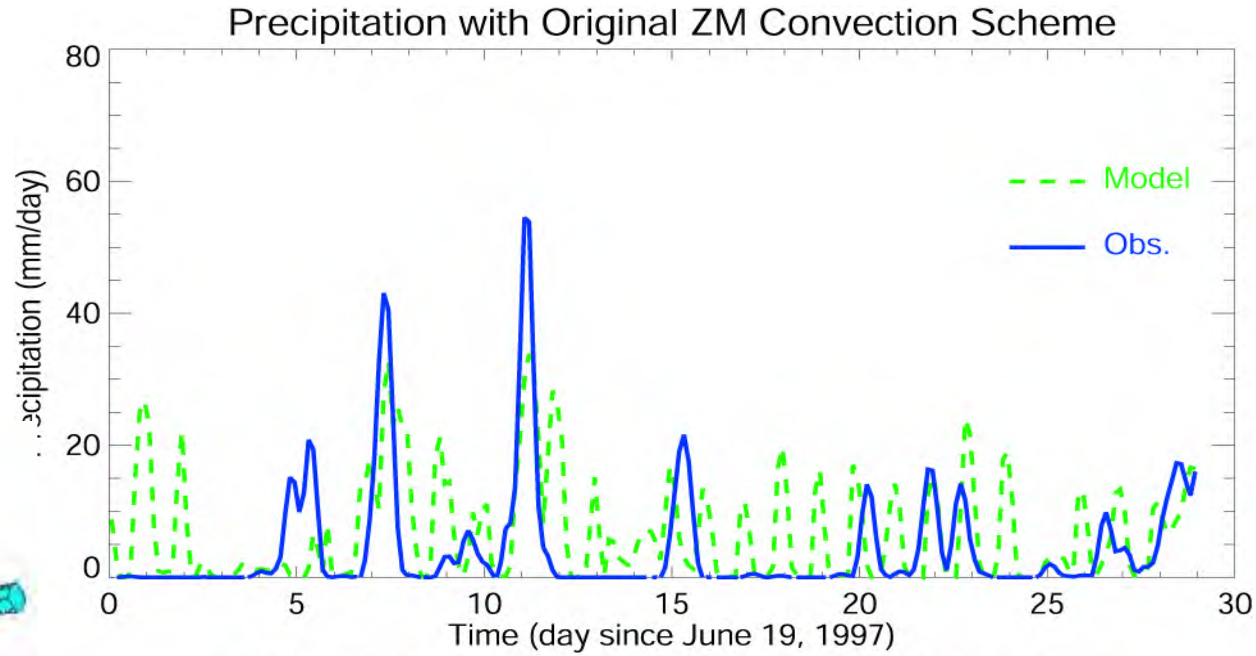
Revised Z-M scheme

deep
convection

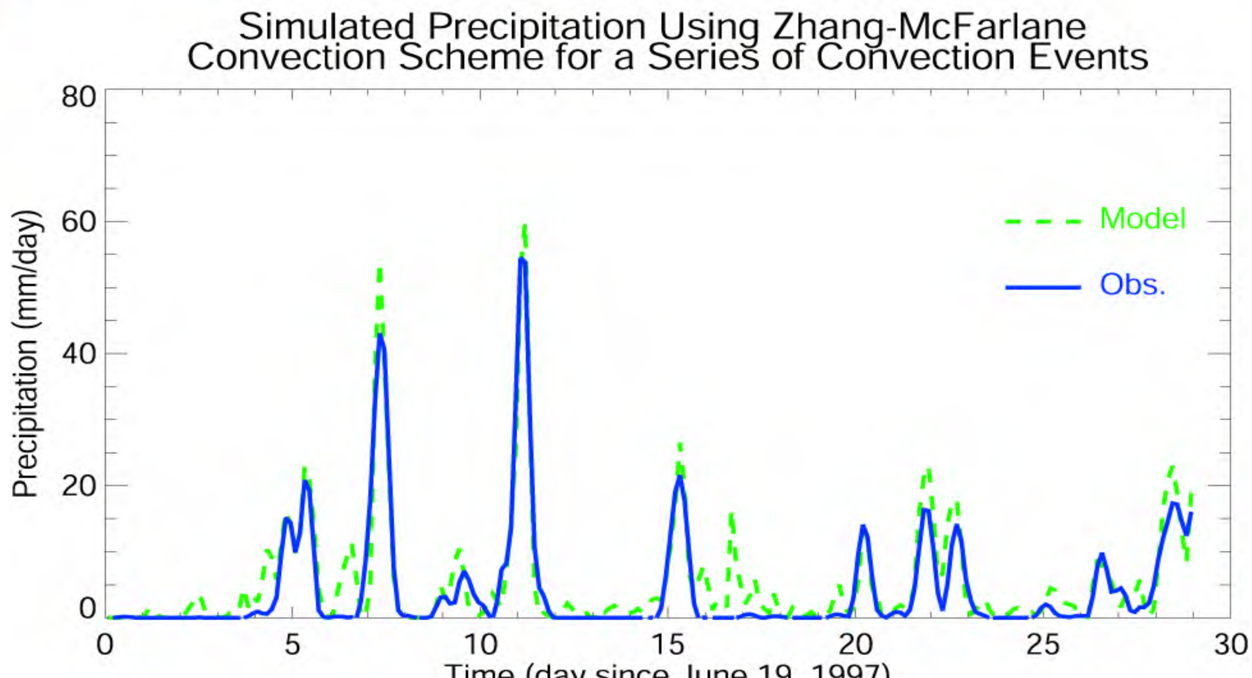


no deep convection

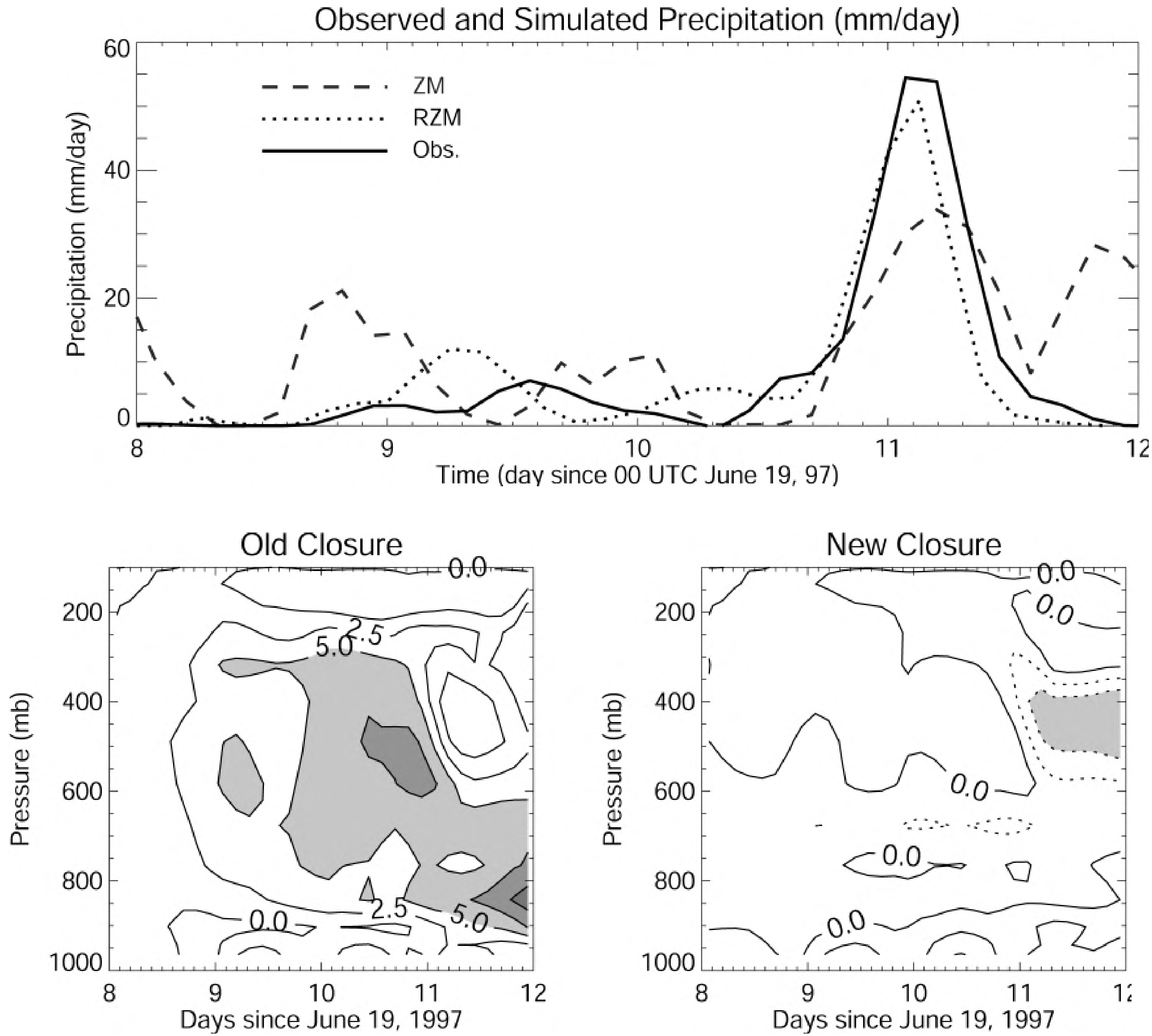




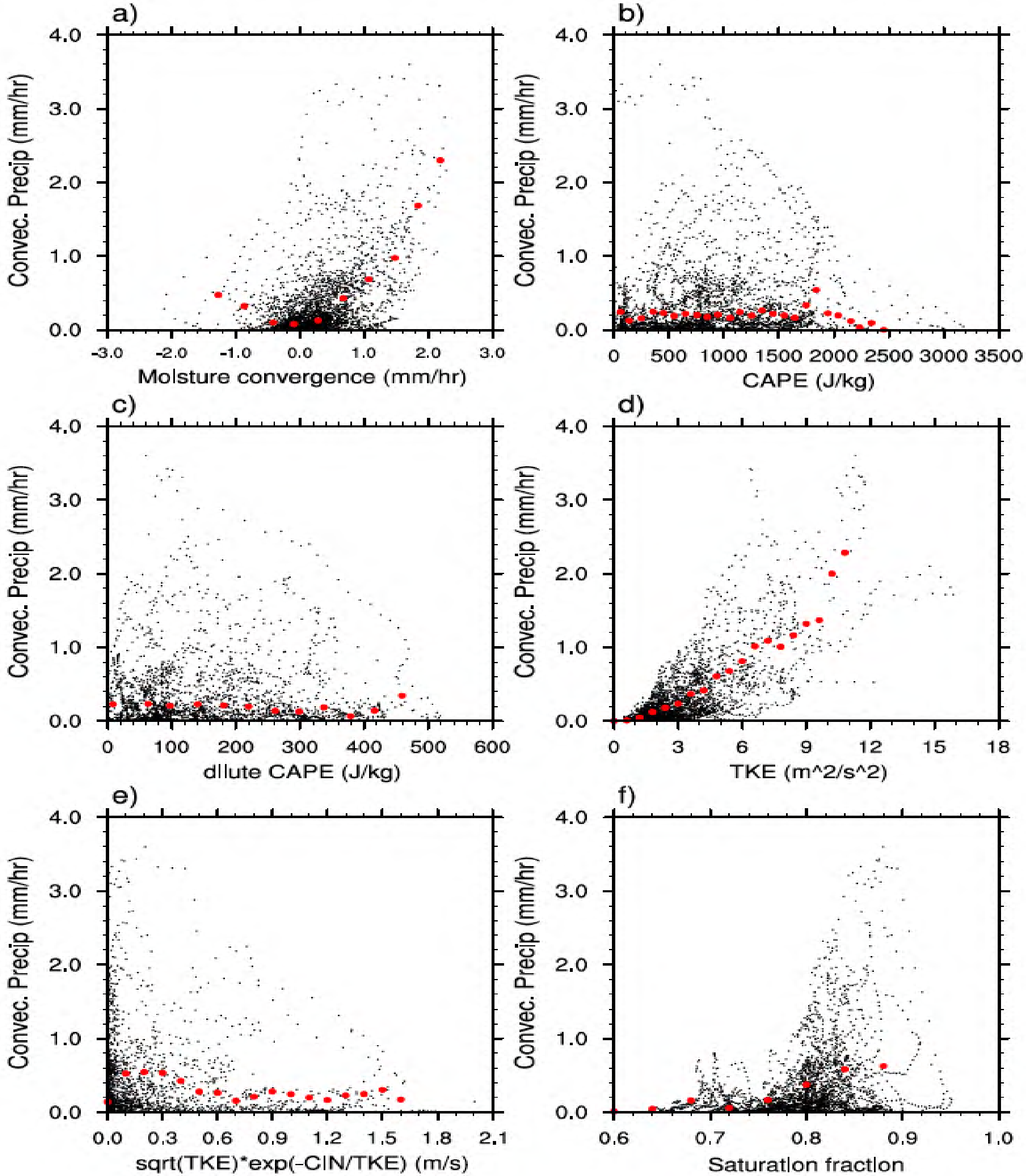
Precipitation with Original ZM Convection Scheme
Precipitation (mm/day)
Time (day since June 19, 1997)
Model
Obs.
0
5
10
15
20
25
30
20
40
60
80



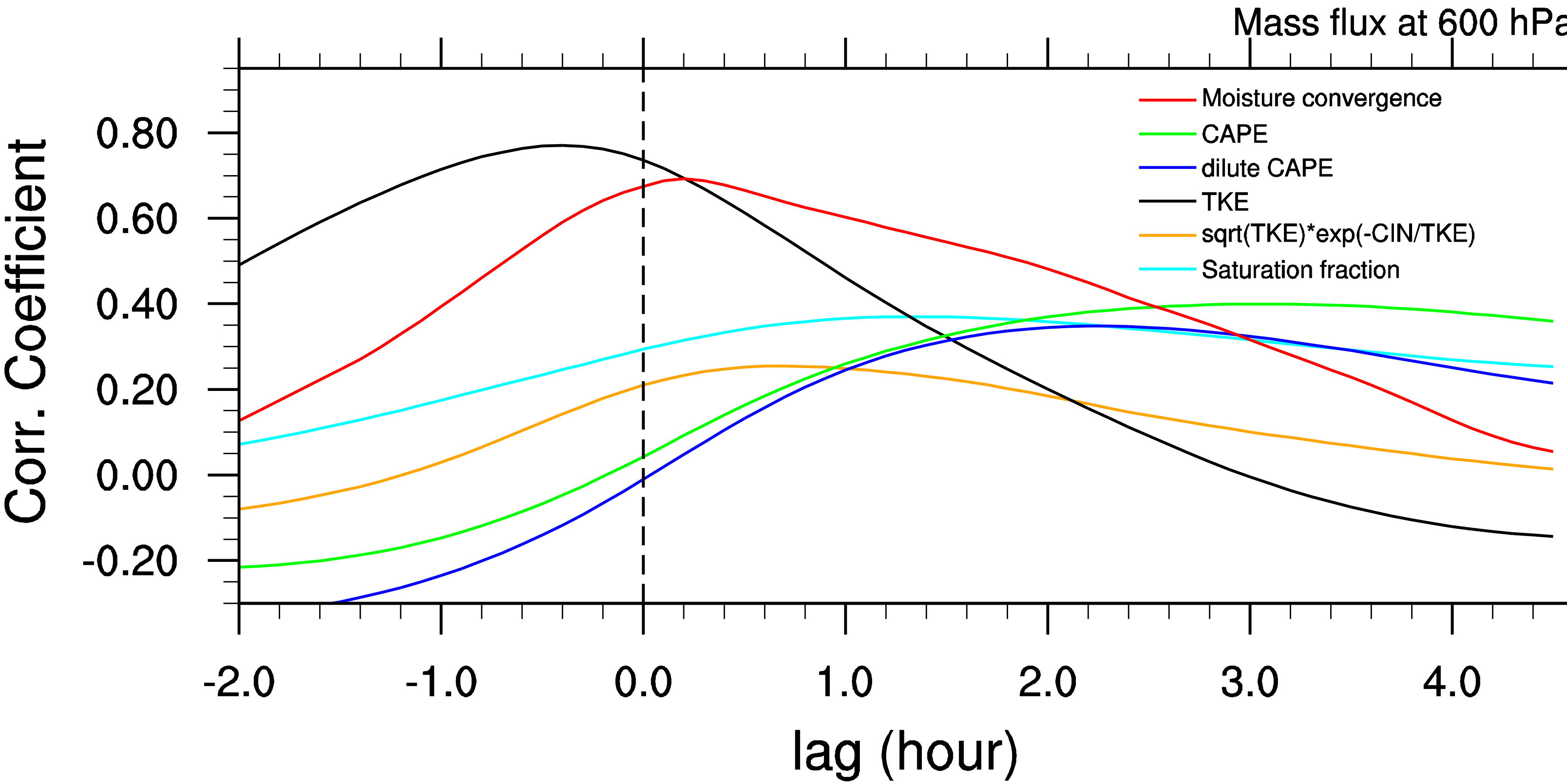
Simulated Precipitation Using Zhang-McFarlane Convection Scheme for a Series of Convection Events
Precipitation (mm/day)
Time (day since June 19, 1997)
Model
Obs.
0
5
10
15
20
25
30
20
40
60
80

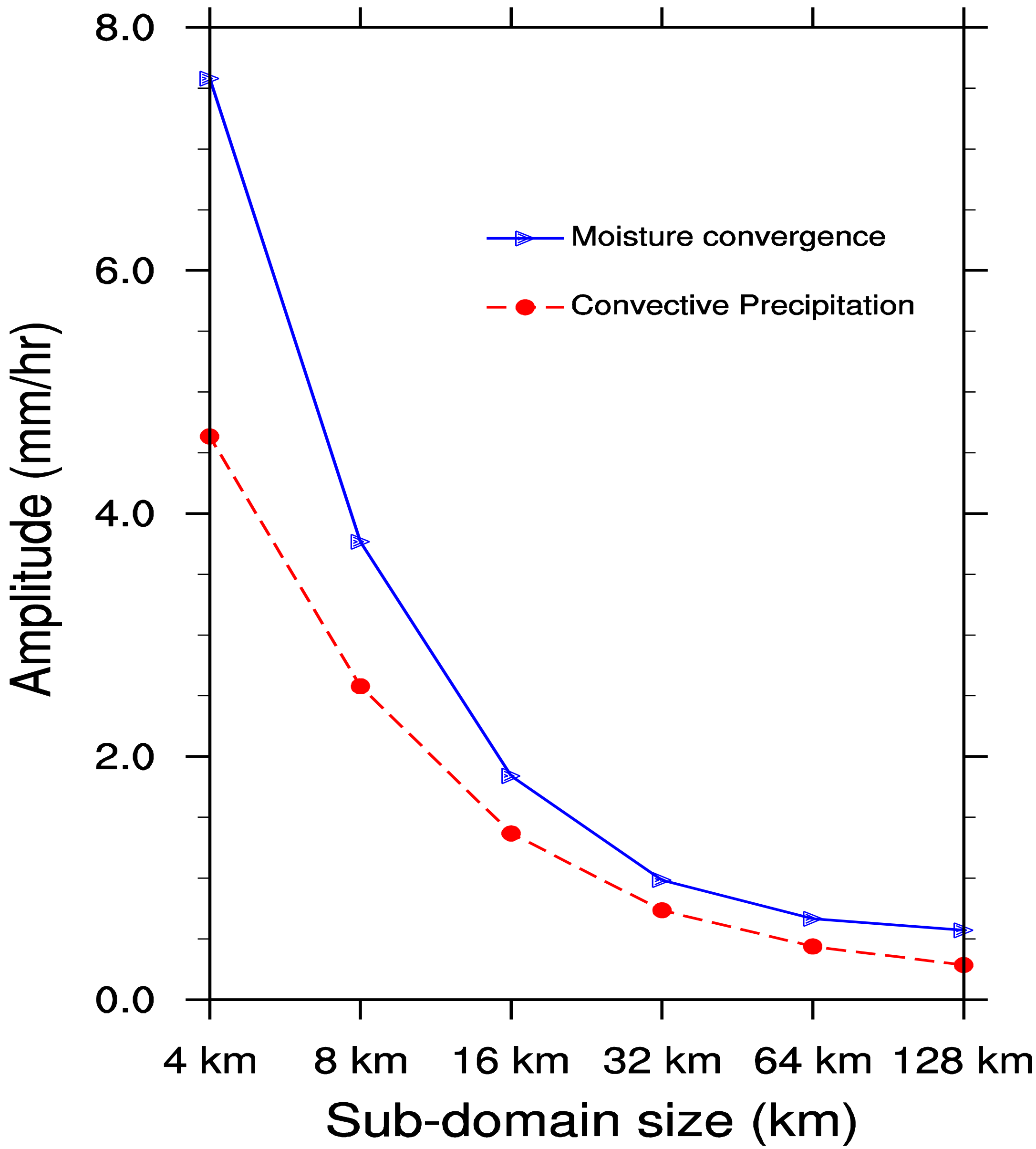


Observed and Simulated Precipitation (mm/day)
ZM
RZM
Obs.
Precipitation (mm/day)
Time (day since 00 UTC June 19, 97)
Old Closure
New Closure
Pressure (mb)
Days since June 19, 1997
0.0
2.5
5.0

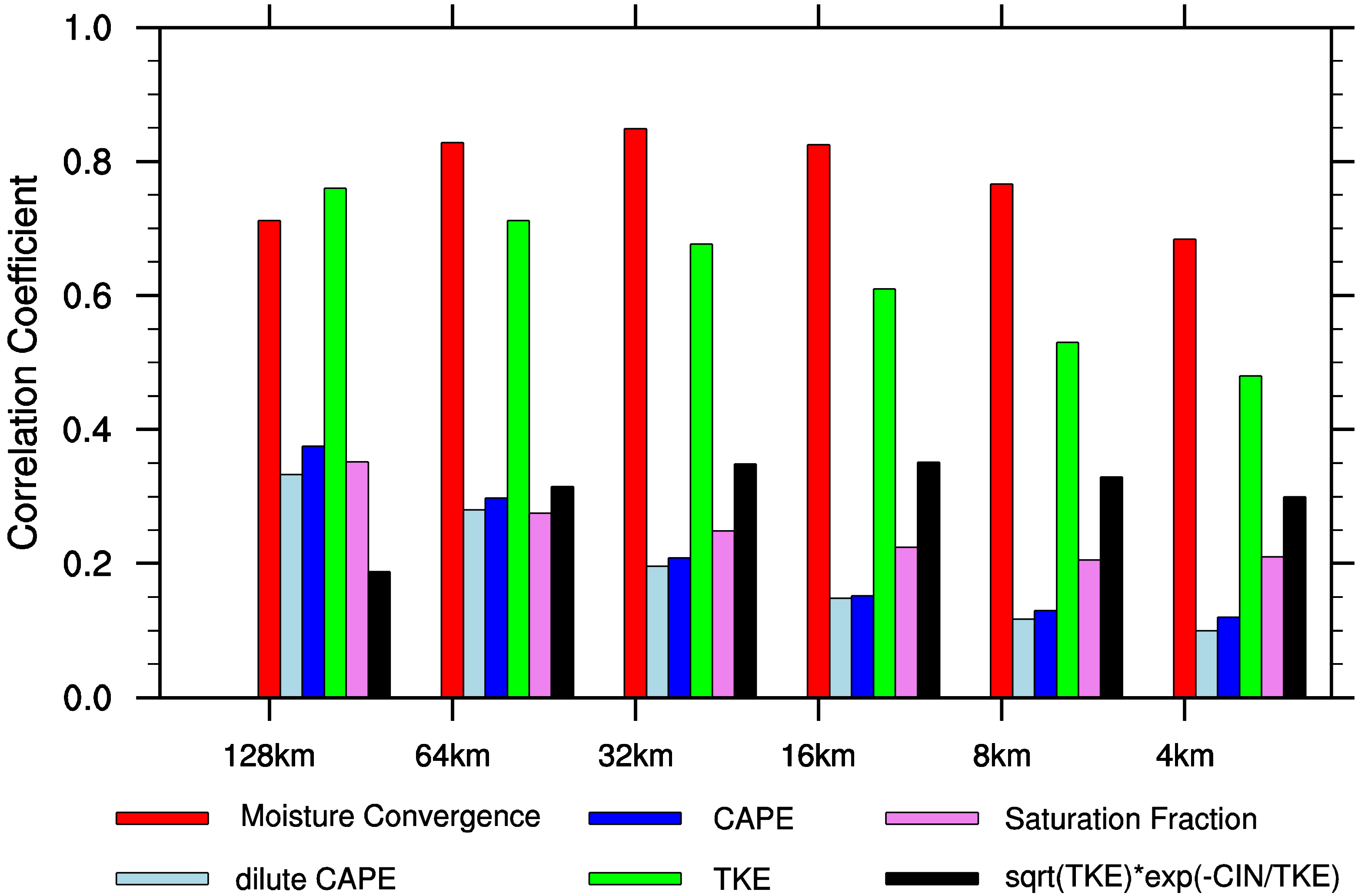


Evaluation of different closures


Mass flux at 600 hPa
Moisture convergence
CAPE
dilute CAPE
TKE
sqrt(TKE)*exp(-CIN/TKE)
Saturation fraction
Corr. Coefficient
0.80
0.60
0.40
0.20
0.00
-0.20
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
lag (hour)



8.0
6.0
4.0
2.0
0.0
Amplitude (mm/hr)
Moisture convergence
Convective Precipitation
4 km
8 km
16 km
32 km
64 km
128 km
Sub-domain size (km)



1.0
0.8
0.6
0.4
0.2
0.0
Correlation Coefficient
128km
64km
32km
16km
8km
4km
Moisture Convergence
CAPE
Saturation Fraction
dilute CAPE
TKE
sqrt(TKE)*exp(-CIN/TKE)

Prognostic Closure

- The above closure conditions are all diagnostic. A serious drawback with diagnostic closures is that they cannot account for the history or memory of convection.
- Pan and Randall (1998) explored a prognostic closure under the framework of the Arakawa-Schubert convection parameterization.
- Instead of assuming a quasi-equilibrium between convective and large-scale processes, they predict column-integrated subgrid scale eddy kinetic energy, which is presumably associated with convective circulation.

The equation is given by

$$\frac{\partial K}{\partial t} = AM_b - \frac{K}{\tau_D}$$

where $K = \frac{1}{2}\int_{Z_s}^{Z_t} \rho(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$ is the total eddy kinetic energy integrated from the surface to the cloud top, A is the cloud work function, M_b is the cloud base mass flux, and τ_D is the dissipation timescale for eddy kinetic energy, a tuning parameter.

Pan and Randall (1998) further related K to M_b through the following assumption

$$K = \alpha M_b^2$$

where α is another tuning parameter.