Modular Ocean Model v6

Overview, discretization, and time-stepping

Presented by Andrew Shao (andrew.shao@noaa.gov)
April 2018, Taiwan Central Weather Bureau

Material from this slide series:
Alistair Adcroft (Princeton), Scott Bachman (NCAR), Robert Hallberg (GFDL), Matthew Harrison (GFDL), Kate Hedstrom (University of Alaska-Fairbanks), Gustavo Marques (NCAR), and Andrew Shao (University of Victoria)

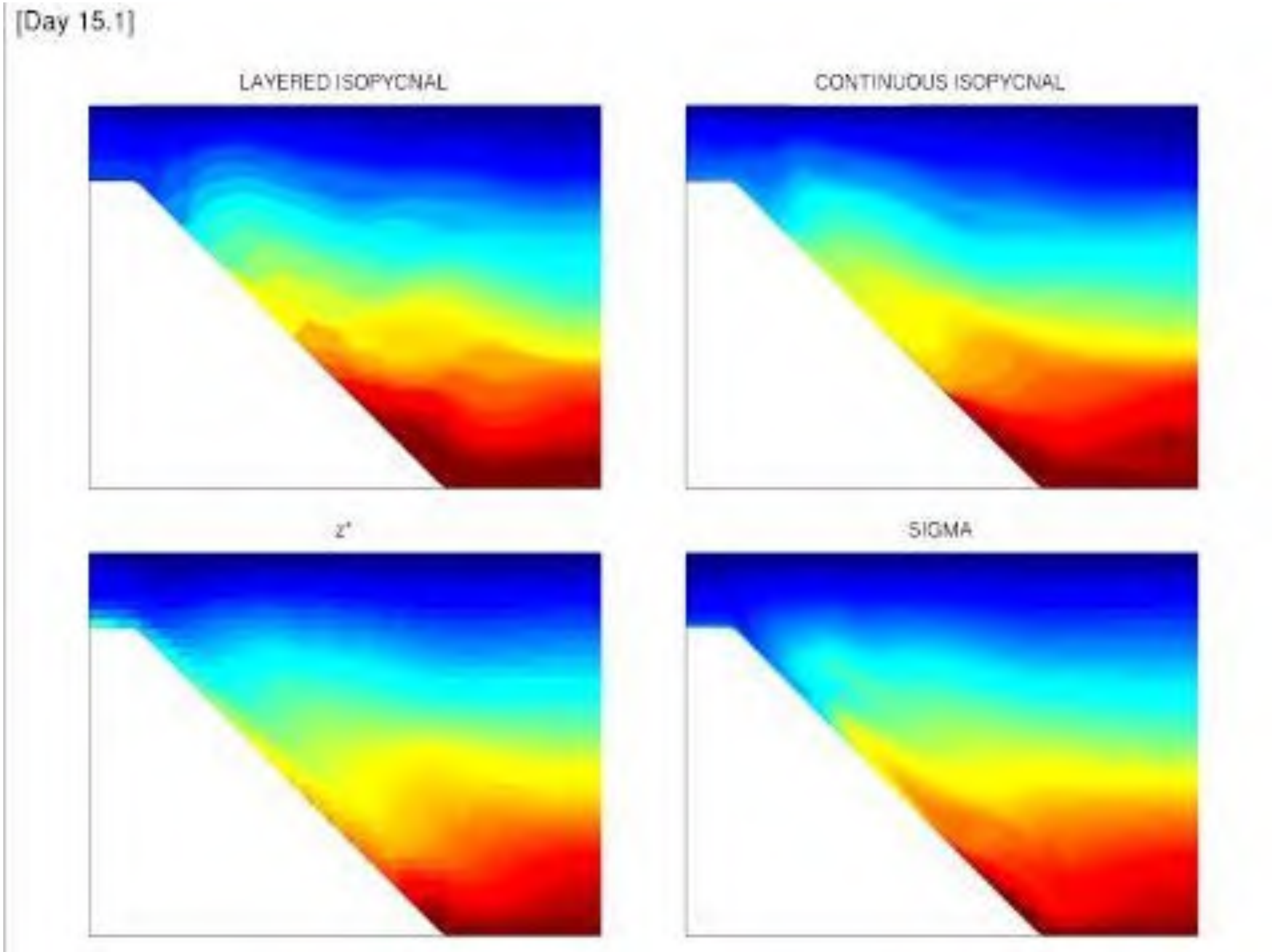
Goals of this presentation:

1. Demonstrate new capabilities of MOM6
2. Introduce the governing equations
3. Describe the fundamental numerics

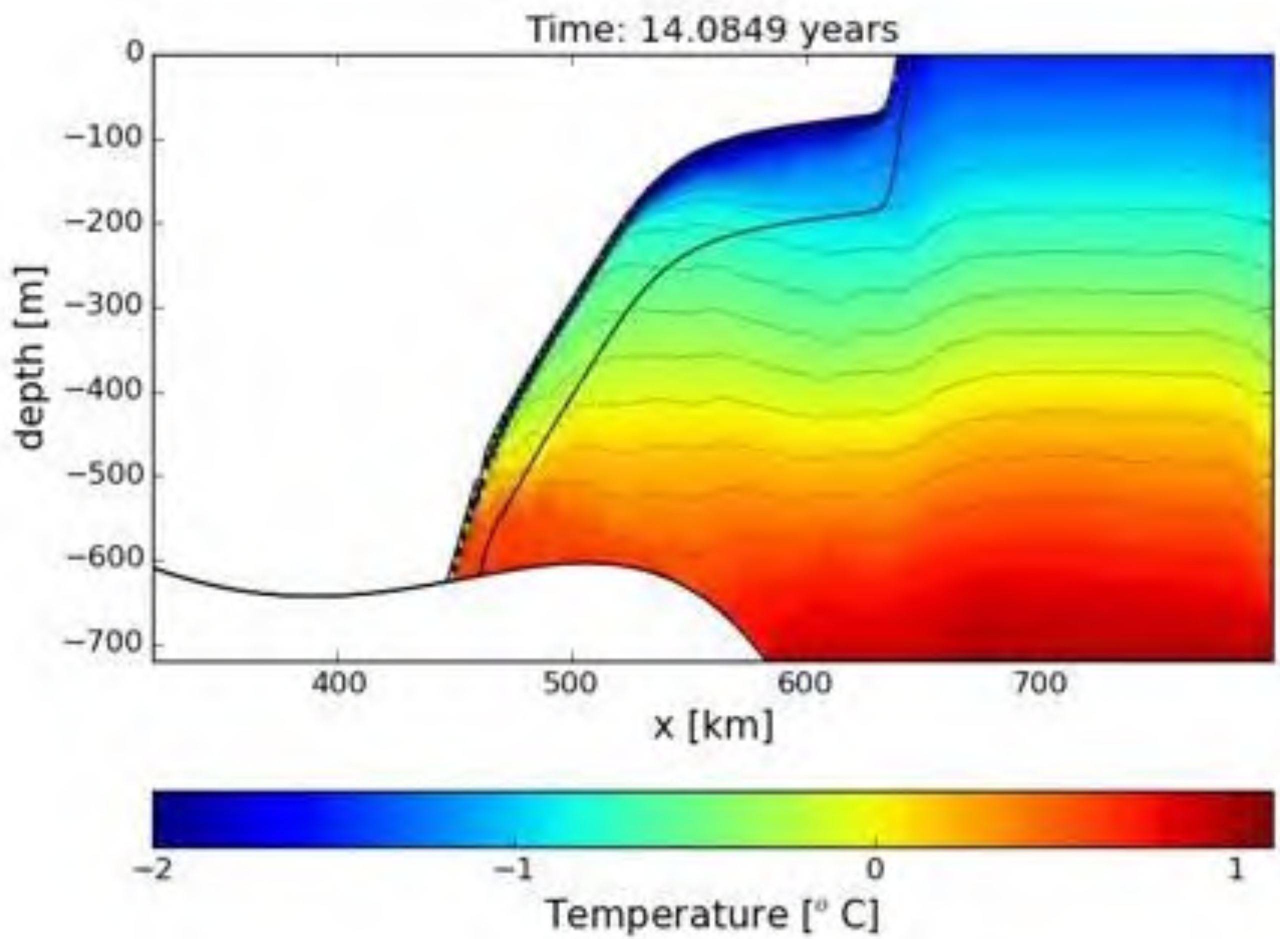
Modular Ocean Model version 6

- Continues a long tradition of ocean model development at NOAA’s Geophysical Fluid Dynamics Laboratory (GFDL)
  - United experiences from MOM, HIM/GOLD, MITgcm
- Improvements over MOM4 and MOM5
  - General-coordinate formulation
  - Wetting and drying
  - Novel parameterizations of sub-gridscale physics
  - Data assimilation
  - C-grid discretization
- Consortium development
  - GFDL
  - National Center for Atmospheric Research (NCAR)
  - National Centers for Environmental Prediction (NCEP)
  - United States Navy
  - Earth System Modeling Group (academic institutions)
  - Consortium for Ocean-Sea Ice Modelling in Australia

Flexible vertical coordinate gives resolution ‘where it’s needed’

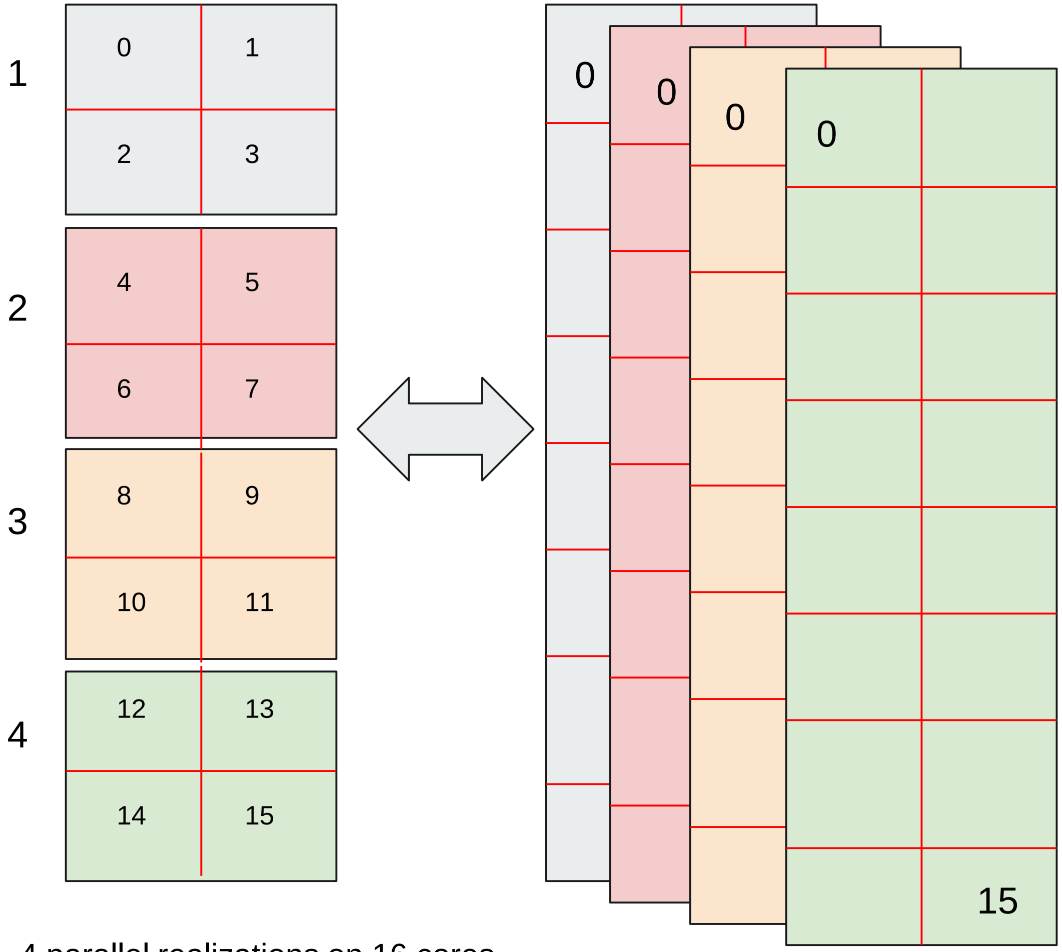


Wetting and drying capabilities (Ice-sheet/ocean demonstration)



Open boundary conditions allow for regional applications

Ongoing development for data assimilation

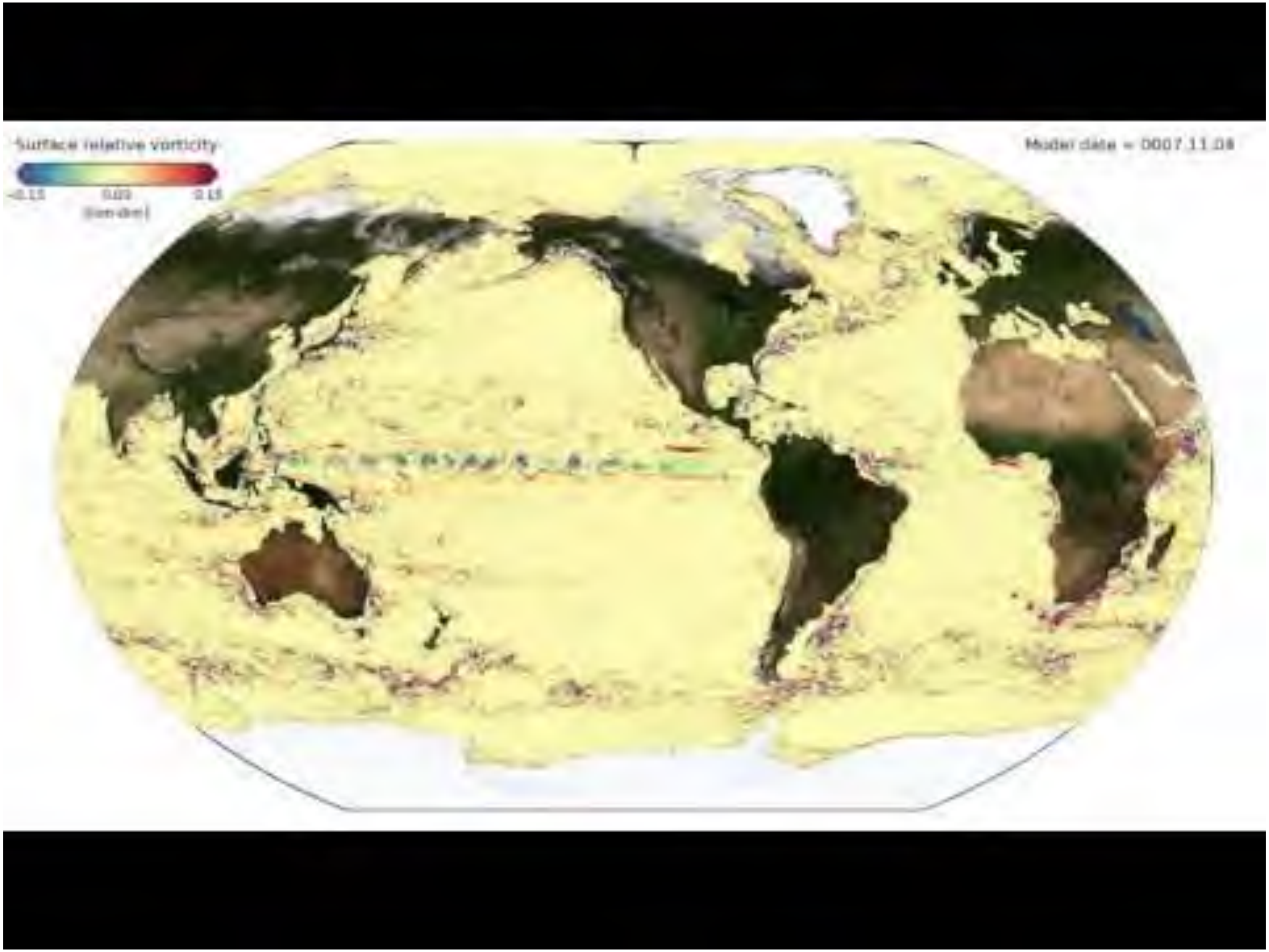


4 parallel realizations on 16 cores

Redistribute for ensemble analysis - each core has copies of all ensemble members

- New driver allows model instances to be run in parallel
  - Instances gathered at analysis time using all PEs or some subset .
  - After the analysis, the updated state or state increments can be redistributed back to the original model decomposition.
- This capability allows for more efficient ensemble analysis in some cases compared to offline methods.

Scale-aware eddy parameterizations for eddy-permitting simulations



MOM6 governing equations are written in **general vertical coordinates** and employ an **arbitrary Lagrangian-Eulerian (ALE)** framework

Simplifying assumptions and implications

- Boussinesq
  - A mean value of density is used in momentum equations except where it affects buoyancy fluxes
  - Explicitly ignores compressibility effects
  - Non-Boussinesq is an option in MOM6
- Hydrostatic
  - Vertical motions are assumed small in vertical momentum equation (no vertical advection of momentum)
- Shallow water
  - Do not consider the curvature of the Earth as a term in momentum equations

The primitive equations in continuous general coordinates: A mathematical description of the ocean

Horizontal momentum: $\rho_0(D_t\vec{u} + f\hat{k} \wedge \vec{u}) + \rho\nabla_r\Phi + \nabla_r p = \nabla \cdot \vec{\tau}$

Vertical momentum: $\rho\partial_r\Phi + \partial_r p = 0$

Continuity: $\partial_t z_r + \nabla_r \cdot z_r\vec{u} + \partial_r z_r\dot{r} = 0$

Temperature: $\partial_t(z_r\theta) + \nabla_r \cdot (z_r\theta\vec{u}) + \partial_r(z_r\theta\dot{r}) = z_r\nabla \cdot Q_\theta$

Salinity: $\partial_t(z_r S) + \nabla_r \cdot (z_r S\vec{u}) + \partial_r(z_r S\dot{r}) = z_r\nabla \cdot Q_S$

$\Phi = gz$

r denotes a general vertical coordinate

$z_r\dot{r}$ is vertical motion relative to coordinate

Integrate ‘r’ coordinate to get thicknesses

Layer thickness: $h = \int_{r_1}^{r_2} z_r \; dr$

Horizontal momentum: $\rho_0(D_t\vec{u} + f\hat{k} \wedge \vec{u}) + \rho\nabla_r\Phi + \nabla_r p = \nabla \cdot \vec{\tau}$

Vertical momentum: $\rho\delta_k\Phi + \delta_k p = 0$

Continuity: $\partial_t h + \nabla_r \cdot h\vec{u} + \cancel{\delta_k z_r}\dot{r} = 0$

Temperature: $\partial_t(h\theta) + \nabla_r \cdot (h\theta\vec{u}) + \partial_r(\cancel{z_r}\theta\dot{r}) = h\nabla \cdot Q_\theta$

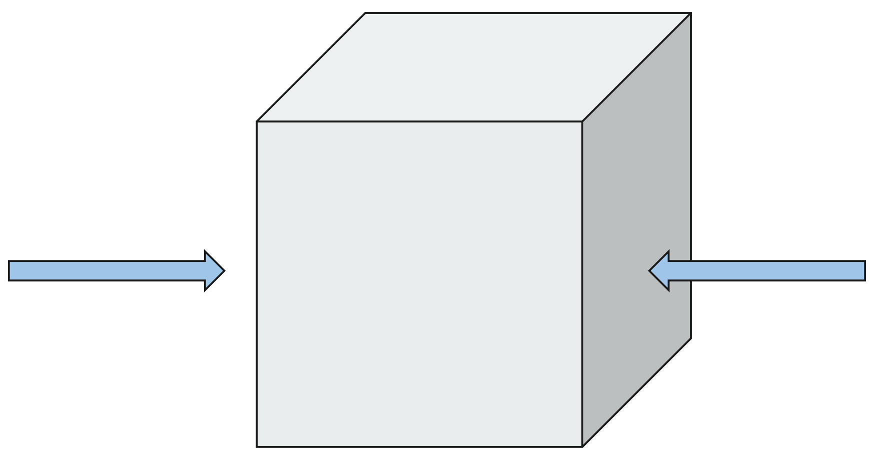
Salinity: $\partial_t(hS) + \nabla_r \cdot (hS\vec{u}) + \partial_r(\cancel{h}S\dot{r}) = h\nabla \cdot Q_S$

Lagrangian limit: $\dot{r} \to 0$

A finite volume approach to solving the primitive equations

- Every element on the grid encompasses a **finite volume**
- Divergence terms in equations replaced by summing fluxes over the faces of the volume

$$\text{Salinity: } \partial_t(hS) + \overbrace{\nabla_r \cdot (hS\vec{u})}^{\text{Divergence}} = h \overbrace{\nabla \cdot Q_S}^{\text{Divergence}}$$



- Total change in salt (tendency) is the convergence of all the fluxes calculated at each face

Discretizations on a staggered C-grid

- Primitive equations have five state variables: u, v, T, S, h
- When discretizing the equations need to consider where each of these variables 'lives' on the grid
- Arakawa [1966] proposed a variety of grids which **stagger** variables: not all variables at the same point
- C-grid trades a computational **velocity** mode for a **Coriolis** mode
  - But can offer better representation of topography for equivalent resolution

Fig 1: Arakawa C-grid of variables around an h-cell with North-East indexing convention

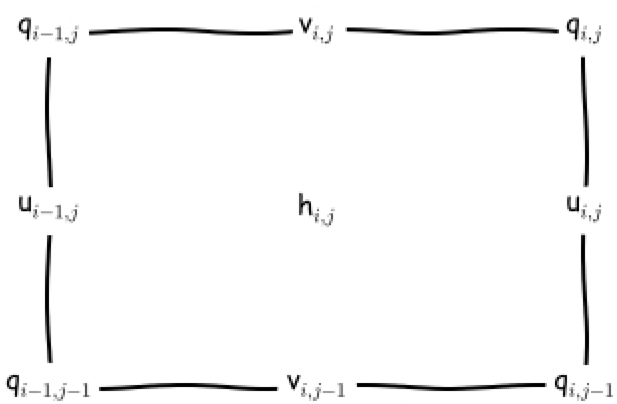
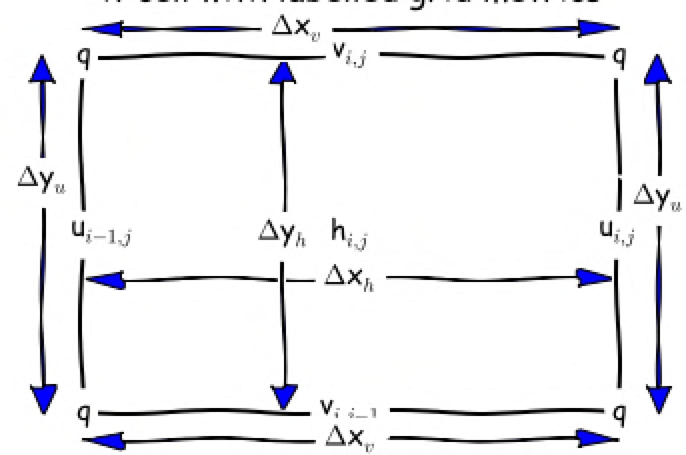


Fig 2a: Arakawa C-grid of variables around an h-cell with labelled grid metrics



Subcycling various parts of the model

Dynamics (900s): Momentum/continuity equations

Coupling (1800s): Fluxes (ice, ocean, atmosphere)

Tracer (3600s)**:** Horizontal Advection and diffusion

Thermodynamics (3600s):ALE/vertical mixing

Primitive equations span multiple timescales

- Ocean physics encompasses a wide variety of timescales
    1. **External gravity wave:** O(100 m/s)
    2. **Mesoscale** O(10 m/s)
- For numerical stability, need a small timestep
- For computational efficiency, want a longer timestep
- CFL condition dictates longest timestep (necessary, but not sufficient condition)

$$C = \frac{U\,\Delta t}{\Delta x} \leq 1$$

$$\Delta t \leq \frac{\Delta x}{U}$$

Timestep satisfying CFL for 1-degree model:

1. **Gravity wave** O(1,000s)
2. **Mesoscale** O(10,000s)

MOM6 splits the primitive equations into a **faster barotropic** (column) piece and a **slower baroclinic** (layer) and integrates in time using a predictor-corrector scheme

Splitting the primitive equations into slow and fast components

$$\rho_0(D_t\vec{u} + f\hat{k} \wedge \vec{u}) + \rho\nabla_r\Phi + \nabla_r p = \nabla \cdot \vec{\tau}$$

$$\rho\delta_k\Phi + \delta_k p = 0$$

$$\partial_t h + \nabla_r \cdot h\vec{u} = 0$$

$$u(x,y,z) = \overline{u(x,y)}^z + u'(z)$$

Barotropic (Vertically averaged motion)

Sum layered continuity equations:

$$\partial_t p_B + \nabla \cdot (\overline{\vec{u}(x,y)}^z p_B) = 0$$

Mass-weighted vertical average of momentum equation

$$\rho_0(\partial_t\overline{\vec{u}}^z + f\hat{k} \wedge \overline{\vec{u}}^z) + \overline{\rho\nabla_r\Phi}^z + \overline{\nabla_r p}^z = \mathcal{F}$$

$$\mathcal{F} = -\frac{1}{p_B}\sum_n \Delta p(\vec{u}_n \cdot \nabla\vec{u}_n) + \sum_n \vec{u}_n\partial_t\frac{\Delta p}{p_B}$$

Baroclinic component (Layer motions)

Layer continuity equation:

$$\partial_t\Delta p' + \nabla \cdot (\vec{u}'\Delta p') + \nabla \cdot \left[\left(\overline{\vec{u}}^{z,t} - \sum_i u_i' \frac{\Delta p_i}{p_B}\right)\Delta_p'\right] = 0$$

Layer momentum equation

$$\begin{aligned}\partial_t\vec{u}' = & \partial_t\overline{\vec{u}}^z - f\hat{k} \wedge (\vec{u}' - \overline{\vec{u}}^z) - (\rho\nabla_r\Phi)' + \nabla_r p' - \vec{u}' \cdot \nabla\vec{u}' + \\ & \frac{1}{p_B}\sum_n \Delta p(\vec{u}' \cdot \nabla\vec{u}') - \sum_n \vec{u}'\partial_t\frac{\Delta p}{p_B}\end{aligned}$$

Integrating barotropic and baroclinic dynamics using a predictor-corrector

$$\rho_0(\partial_t \overline{\vec{u}}^z + f\hat{k} \wedge \overline{\vec{u}}^z) + \overline{\rho \nabla_r \Phi}^z + \overline{\nabla_r p}^z = \mathcal{F}$$

$$\mathcal{F} = -\frac{1}{p_B} \sum_n \Delta p(\vec{u}_n \cdot \nabla \vec{u}_n) + \sum_n \vec{u_n} \partial_t \frac{\Delta p}{p_B}$$

1. Integrate barotropic equations over a number of short timesteps (predicted)
   a. Right hand-side kept constant
   b. Store time averaged transports
2. Predict total velocities and layer thicknesses using averaged barotropic transports in baroclinic equations
3. Integrate the barotropic equations again, but with new right-hand-side (corrected)
   a. Store time averaged transports

The more time steps used in Step 1 and Step 3 trade off accuracy for damping.

Vertical dynamics handled by ALE regridding remapping framework

- Up to this point have only considered the horizontal part of the physics.
- Primitive equations use the Lagrangian limit for vertical motions

$$\text{Horizontal momentum: } \rho_0(D_t\vec{u} + f\hat{k} \wedge \vec{u}) + \rho\nabla_r\Phi + \nabla_r p = \nabla \cdot \vec{\tau}$$

$$\text{Vertical momentum: } \rho\delta_k\Phi + \delta_k p = 0$$

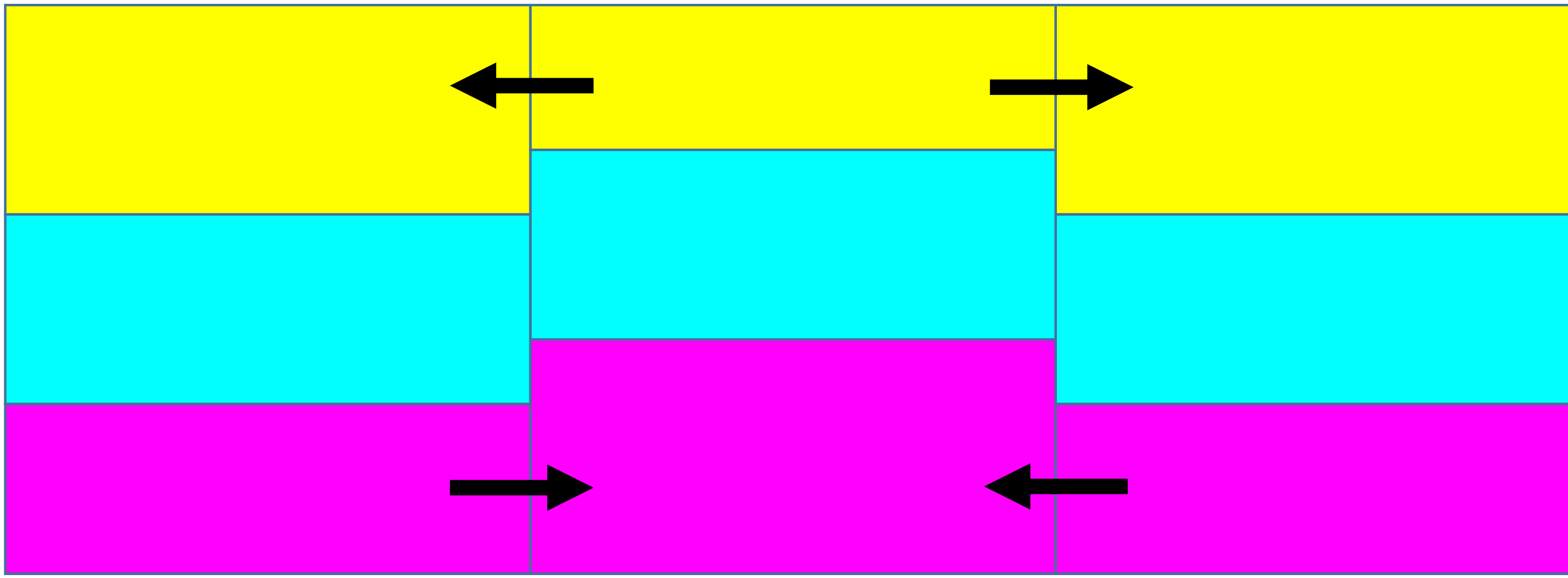
$$\text{Continuity: } \partial_t h + \nabla_r \cdot h\vec{u} + \delta_k z_r\dot{r} = 0$$

$$\text{Temperature: } \partial_t(h\theta) + \nabla_r \cdot (h\theta\vec{u}) + \partial_r(z_r\theta\dot{r}) = h\nabla \cdot Q_\theta$$

$$\text{Salinity: } \partial_t(hS) + \nabla_r \cdot (hS\vec{u}) + \partial_r(hS\dot{r}) = h\nabla \cdot Q_S$$

- Momentum and continuity give changes in layer thickness (similar to isopycnal coordinate models)
- **Key reinterpretation:** Vertical advection is always defined relative to a given grid
- Regridding/remapping takes the place of an explicit advection routine and has **no limit on CFL**

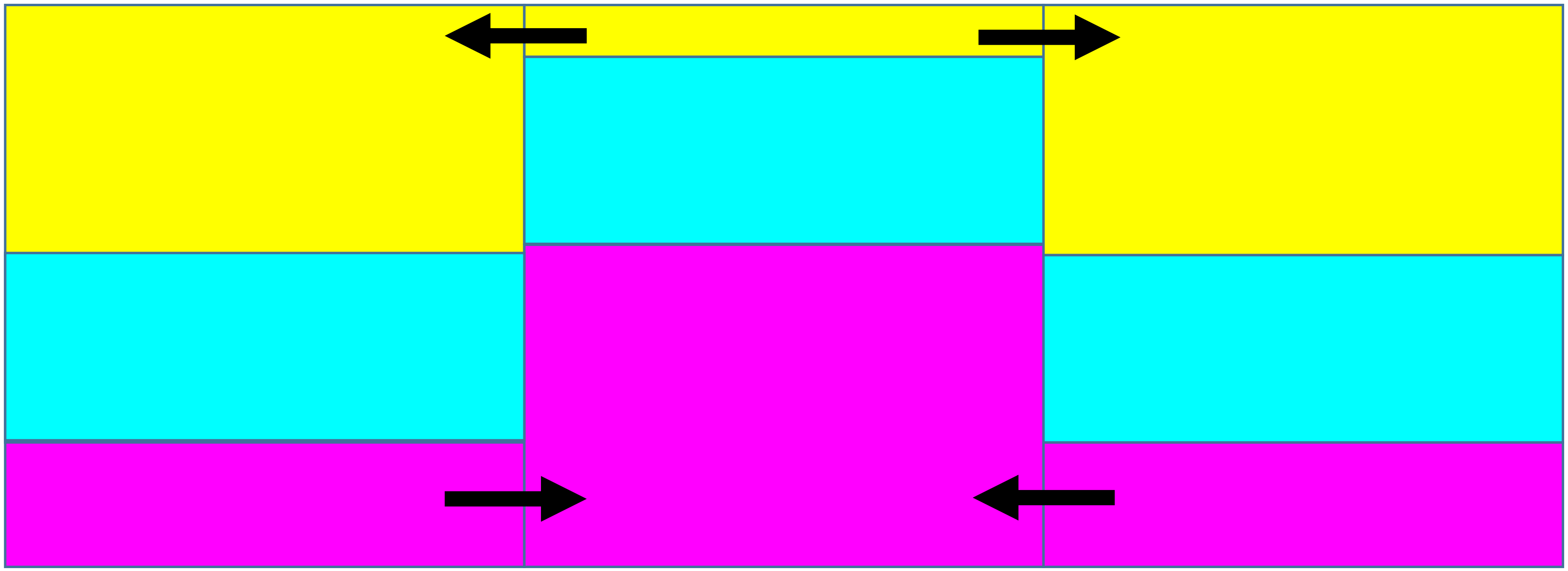
Evolve layers with divergence of lateral flow



$$h^{n+1/3} = h^n - \Delta t \nabla \cdot \left(h^n \boldsymbol{u}^{n+1/3} \right) = 0$$

- Thicknesses are updated based on convergence of lateral transports
- Tracers are still uniform layer by layer, so no net flux of tracer

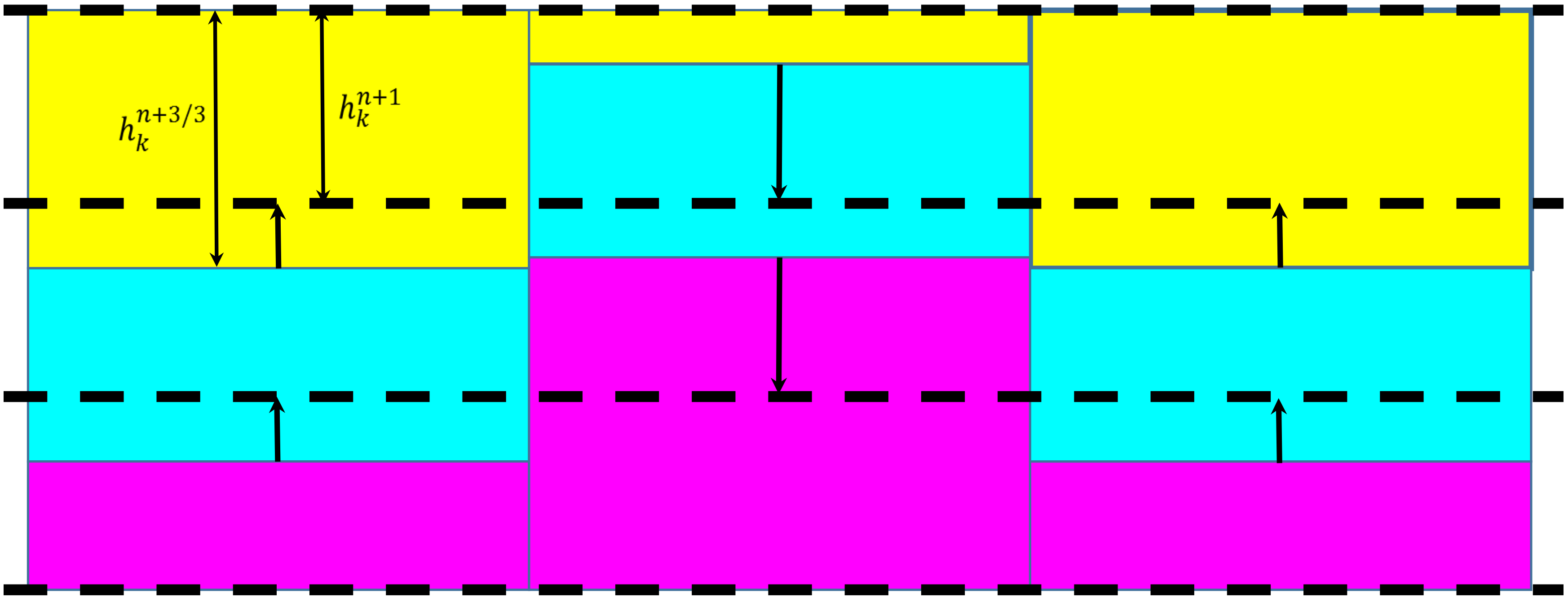
Layers after three dynamics steps



$$h^{n+3/3} = h^{n+2/3} - \Delta t \nabla \cdot \left(h^{n+2/3} \boldsymbol{u}^{n+3/3} \right) = 0$$

- Vertical grid has evolved in a Lagrangian way (thicknesses change)
- Significantly distorted from the original, equally-spaced grid

Arbitrary Lagrangian-Eulerian regridding step

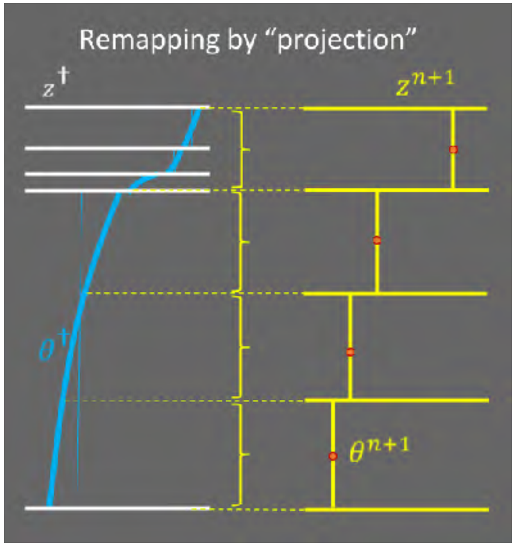


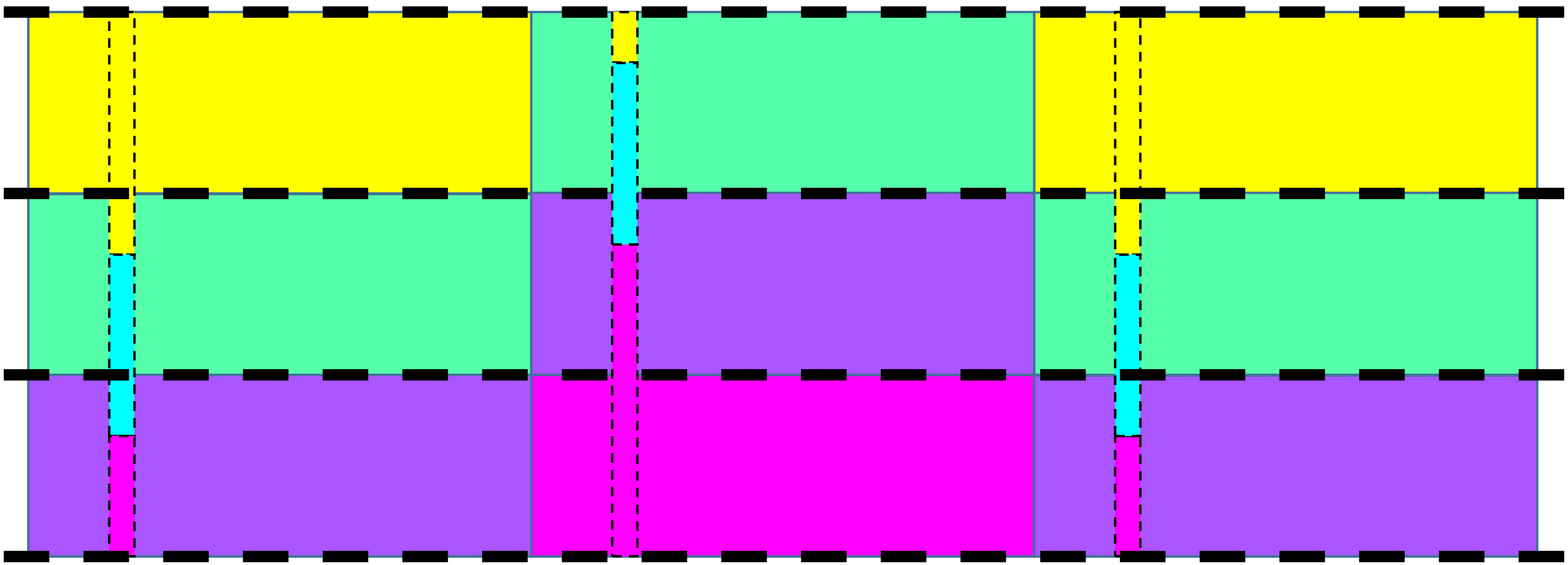
- Any new grid can be imposed (z*, terrain-following, isopycnal, hybrid)
- State-dependent coordinates are difficult, but possible

Polynomial representations of tracers allow conservative remapping between grids

- Finite volume formulation means that tracer values are volume-averaged quantities
- Free to choose a subcell structure
  - Consistent
  - Monotonic
  - No new extrema
- Piecewise polynomials
  - Constant
  - Linear
  - Quadratic
  - Quartic
- Integrate polynomials to project variables onto the new grid

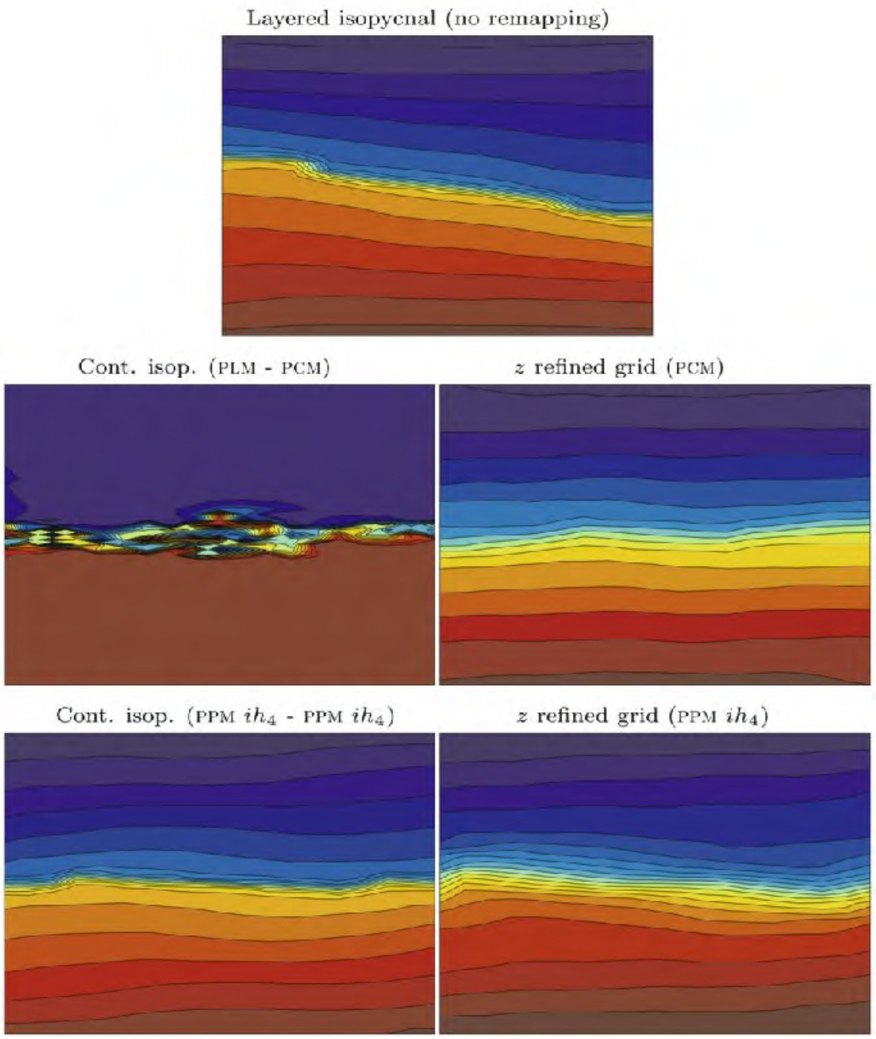
Final state of the layers after regridding/remapping





Choice of vertical coordinate and the order of ALE scheme dictates representation of physics

- Idealized internal wave
  - Without mixing, stratification maintained
- Coordinate
- Stack layered (non-ALE)
  - Maintains stratification
  - Most ‘physical’
- Continuous isopyncal
  - Fails for low-order ALE
  - Maintains strong stratification for higher-order
- Z-like
  - Diffuses pycnocline
  - Required refinement where stratification known to be high



Summary of MOM6 numerics

- Formulation and discretization of the primitive equations
  - Can choose any variety of vertical coordinates
    - Easily evaluate which coordinate is appropriate for a given application
  - Time steps for various components may be chosen separately for computational efficiency
- Recently developed capabilities enable new applications
  - Conservative wetting and drying
    - Ice sheets
    - Coastal regions?
  - Open boundary conditions
    - Regional modeling
  - Resolution-aware parameterizations
    - Models across a range of scales
- Open, consortium-based development
  - Encourages broad user base
  - Reduces likelihood of redundant development
- Next topics
  - Parameterizing subgrid-scale physics